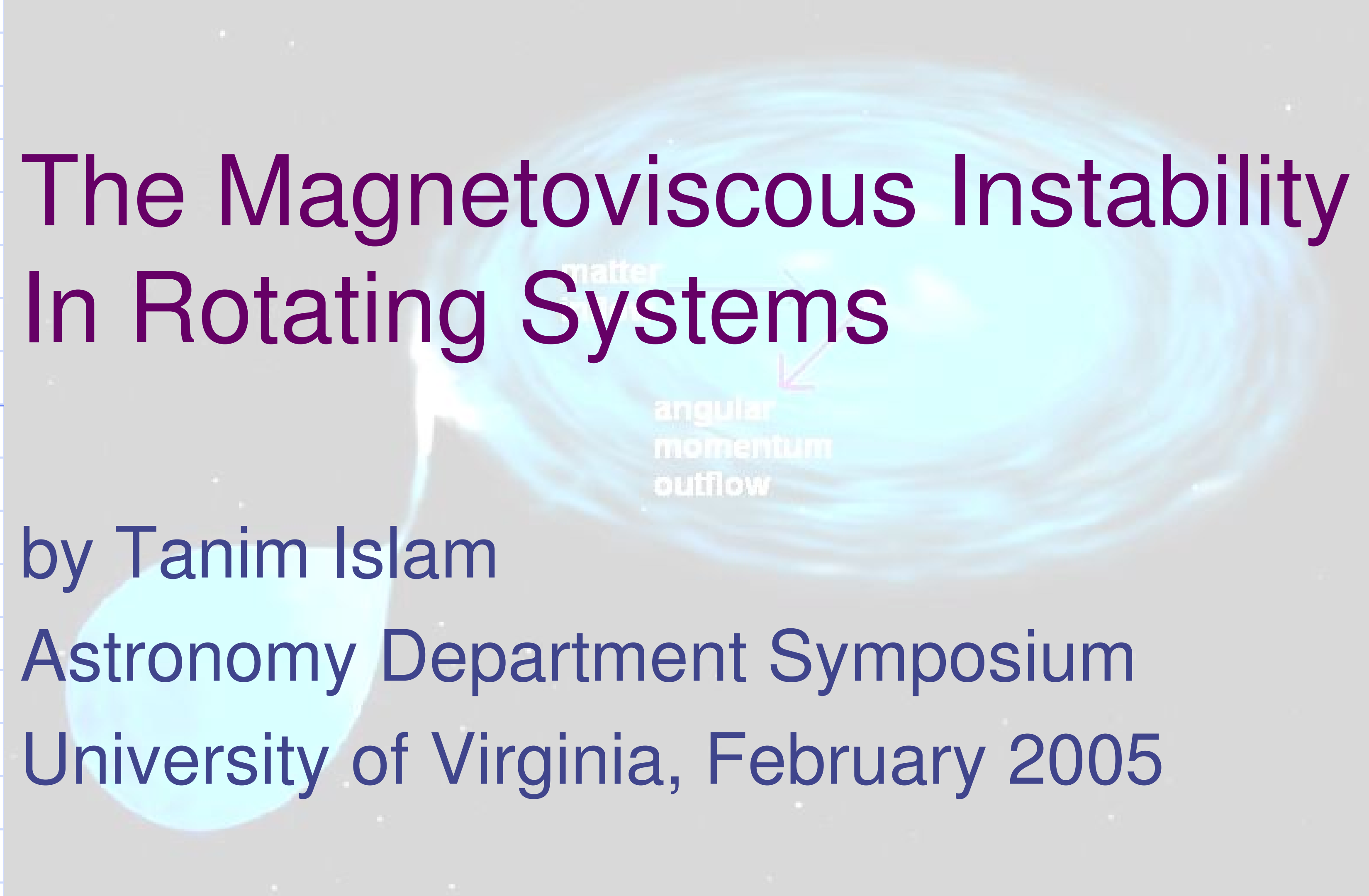
The Magnetoviscous Instability In Rotating Systems



by Tanim Islam
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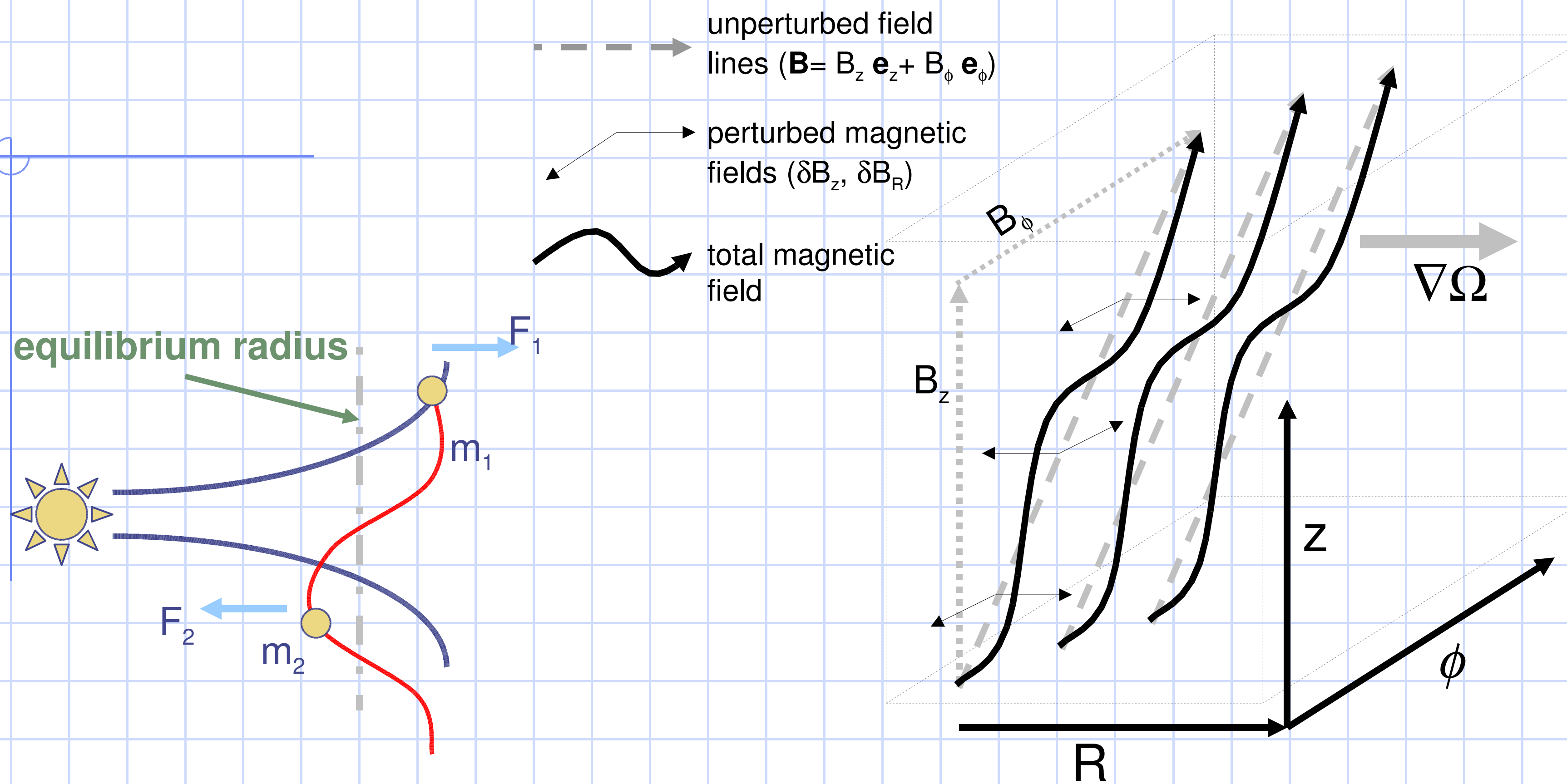
The Magnetorotational Instability

- First discovered by Velikhov[1] and Chandrasekhar[2], magnetized disks with decreasing outwards **angular velocity** Ω rather than decreasing outwards **angular momentum** ΩR^2 (stability criterion for hydrodynamic disks).
- Instability grows at the rate of Ω at wavelengths much **smaller** than the length scale of the unstable fluid.
- Balbus and Hawley[3] demonstrated the global applicability of the MRI to accretion disks.
  - Development into a magnetic turbulent flow within the accretion disk.
  - magnetic pressures saturate at levels ~ gas (thermal) pressure.
  - Numerical simulations gave $\alpha < 1$ (with a viscosity given by $\eta_v = \alpha c_s H$>>collisional viscosities), consistent with observed accretion disks fitted (luminosity, spectra) to α models of accretion disks.[4]

[1] Sov. Phys. JETP **36**, 995 (1959).
[2] Proc. Nat. Acad. Sci. USA **46**, 53 (1960).
[3] Ap. J. **376**, 214 (1991)
[4] N. Shakura and R. Sunyaev, A & A **24**, 337-355 (1973).

Schematic Model of MRI



- Points on a magnetic field line are forced to corotate (same Ω)
- The points further out from the equilibrium tend to accelerate outward, while points inside accelerate inwards
- This is all quenched at small enough wavelengths due to the "springiness" of magnetic tension

Magnetoviscous Instability (MVI)

- Magnetized plasma in which the magnetic field is too weak that, at sufficiently large wavelengths a few orders of magnitude smaller than the disk height **H**, that the magnetic field does not accelerate the fluid (***no magnetic tension/compression forces as in the MRI***).
- However, the magnetic field (however weak) is strong enough ($\nu_{ii} < \Omega_{ci}$, where ν_{ii} is the collision freq. and Ω_{ci} is ion cyclotron frequency) to anisotropize the viscosity – to force the viscosity to lie along the magnetic field line.[1,2]
- The resulting instability, if magnetic tension forces are ignored (as can be done for sufficiently long wavelengths $\lambda > \Omega/v_A$, where v_A is the Alfven velocity or the speed of magnetic tension disturbances), asymptotically reaches a maximum growth rate at wavelengths $\lambda \sim (\eta_\nu/\Omega)^{1/2}$.
- Physically, for the MRI fluids were tethered to each other through **magnetic** force; through the MVI, fluids are tethered to each through an **anisotropic viscous** force (which is aligned on magnetic field lines).

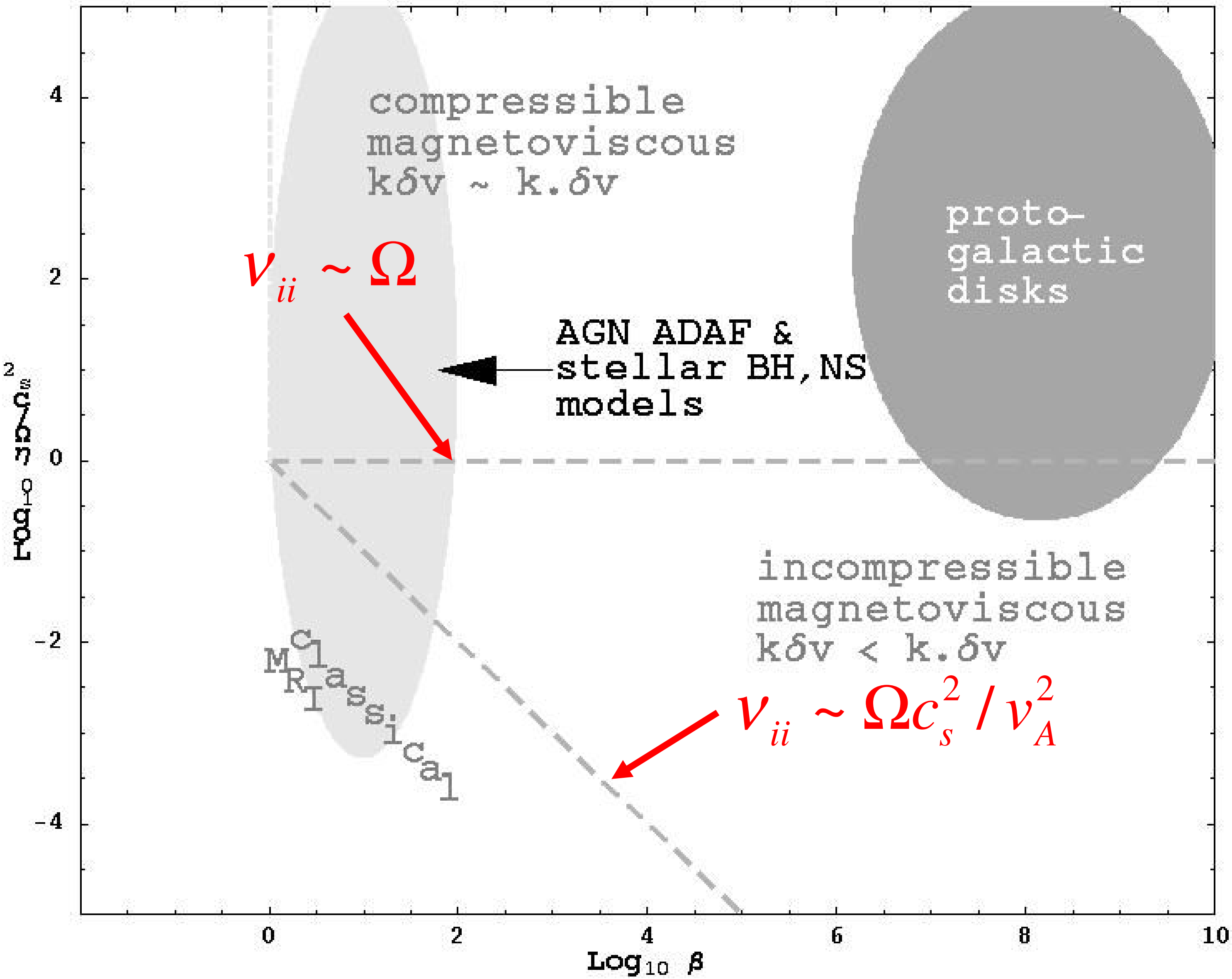
[1] S. I. Braginskii, Rev. of Plasma Physics Vol. 1 (New York: Cons. Bureau, 1965)

[2] J. Huba, NRL Plasma Formulary (Washington DC: NRL, 2002)

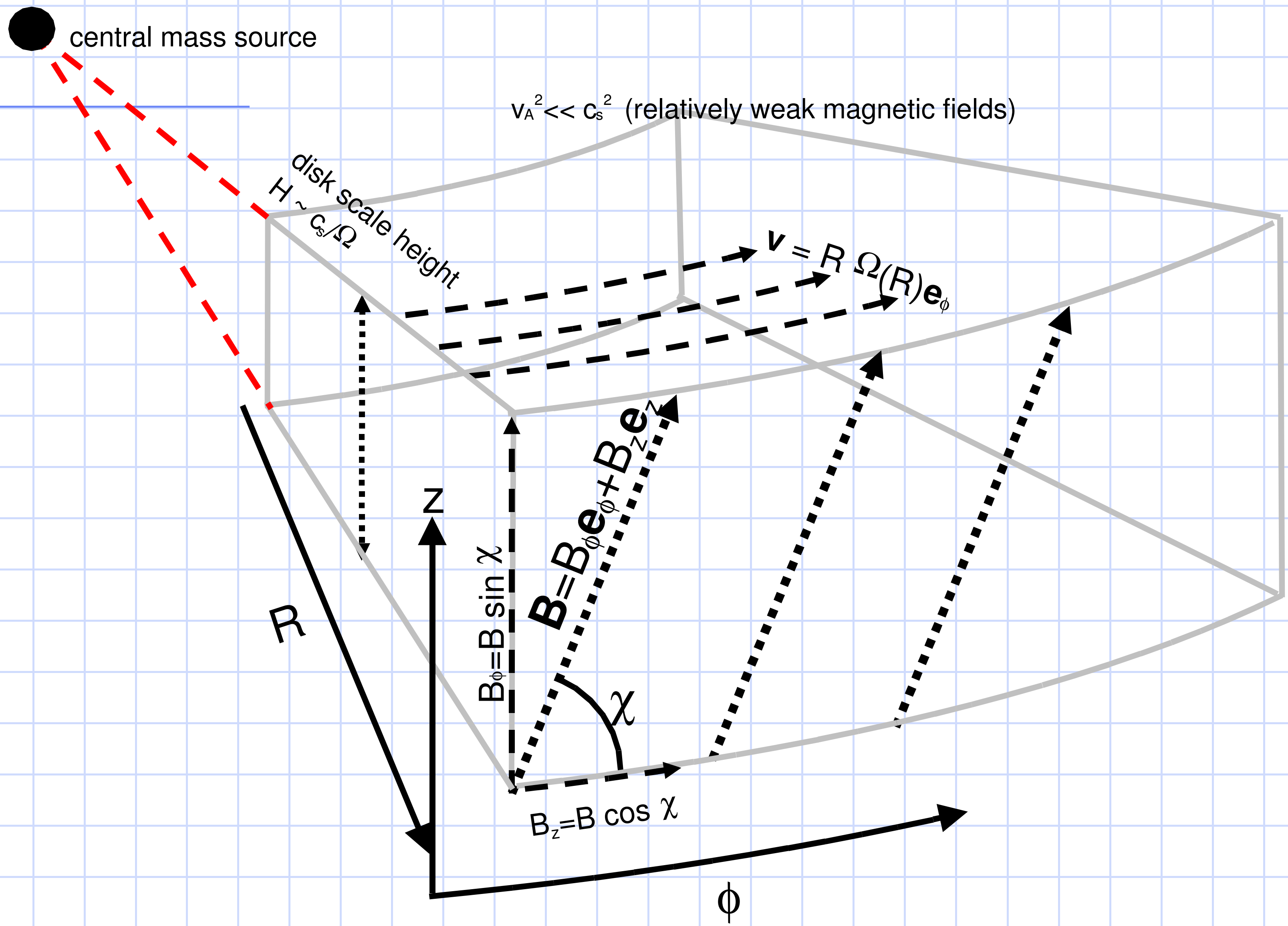
Justification for Study of the MVI

- Certain classes of rotating astrophysical objects are unstable to these modes – those that are characterized by very dilute plasmas and relatively weak magnetic fields.
  - protogalactic disks – amplification of weak magnetic fields.
  - RIAFs – very hot (ion temperatures ~ 10^{12} K), dilute, optically thin and nonradiative plasma around compact objects.
  - Certain classes of rotating astrophysical objects are unstable to these modes – those that are characterized
- Magnetic turbulence due to nonlinear development of the MVI should (naturally) occur at length scales much larger than the natural turbulence length scale in MRI.

Astrophysical Objects Unstable to the MVI



Equilibrium Disk



Details of Stability Analysis

- axisymmetric instabilities, $\delta a \propto \exp\left(ik_z z + ik_R R + \Gamma t\right)$
- where δa is perturbed quantity, Γ is growth rate, and k_R and k_z are radial and vertical wavenumbers.
- **Boussinesq approximation – incompressible instabilities.**
- WKB (wave) approximation, examining wavelengths < H (k H > 1).
- equilibrium rotating disk is one in which:
  - the magnetic field is nonradial, $\mathbf{B}=B_R\mathbf{e}_R+B_z\mathbf{e}_z$ (no B-field time dependance)
  - the equilibrium velocity is rotational $\mathbf{v}= R\Omega\,\mathbf{e}_\phi$
  - the magnetic field is highly subthermal $v_A^2<<c_s^2$

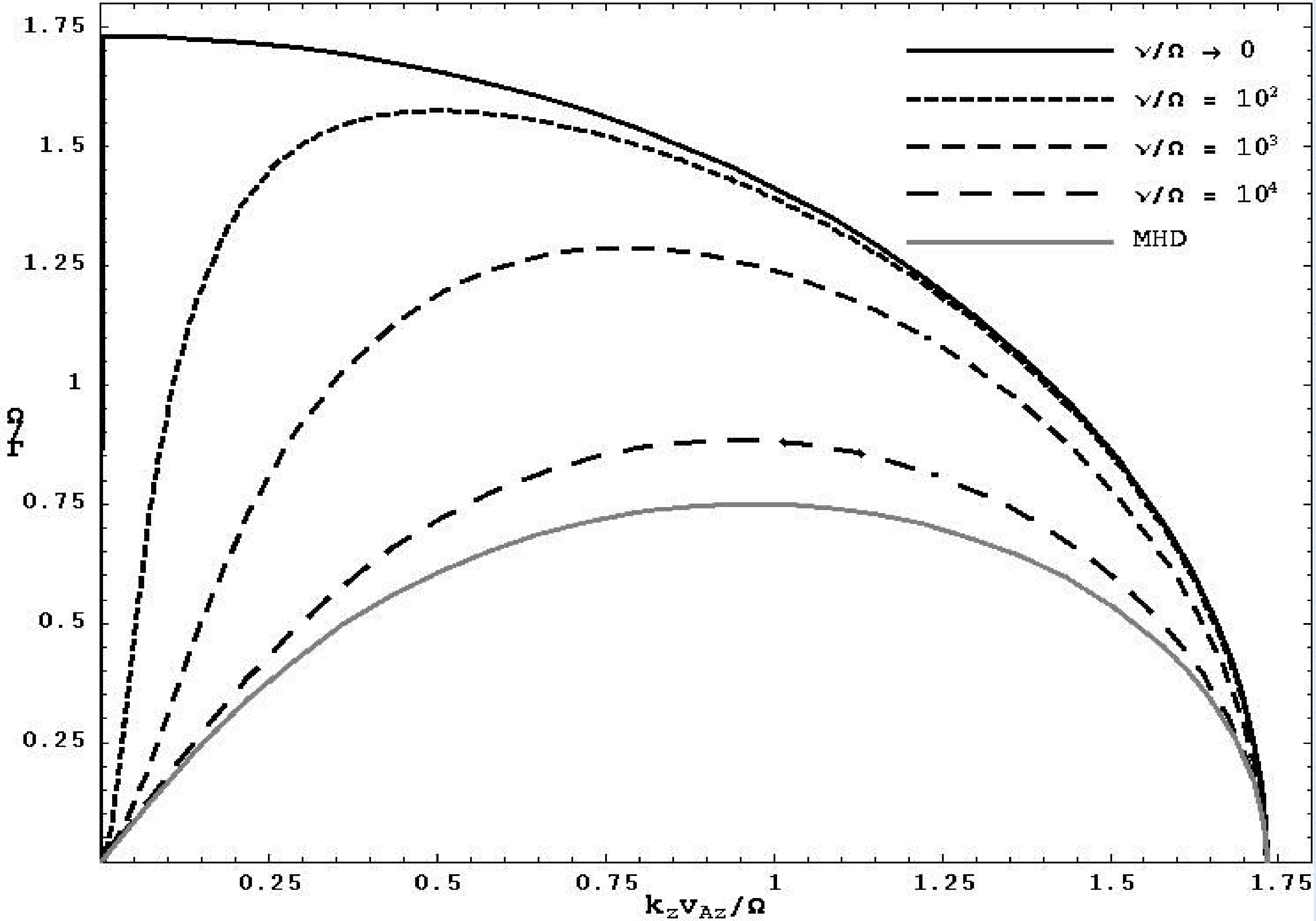
- the following are the list of normalizations (of wavenumber, growth rate, viscous diffusion coefficient) as well as useful parameters to characterize the instability

$$\hat{\mathbf{k}} = \mathbf{k}H = \mathbf{k}c_s/\Omega$$

$$\hat{\Gamma} = \Gamma/\Omega$$

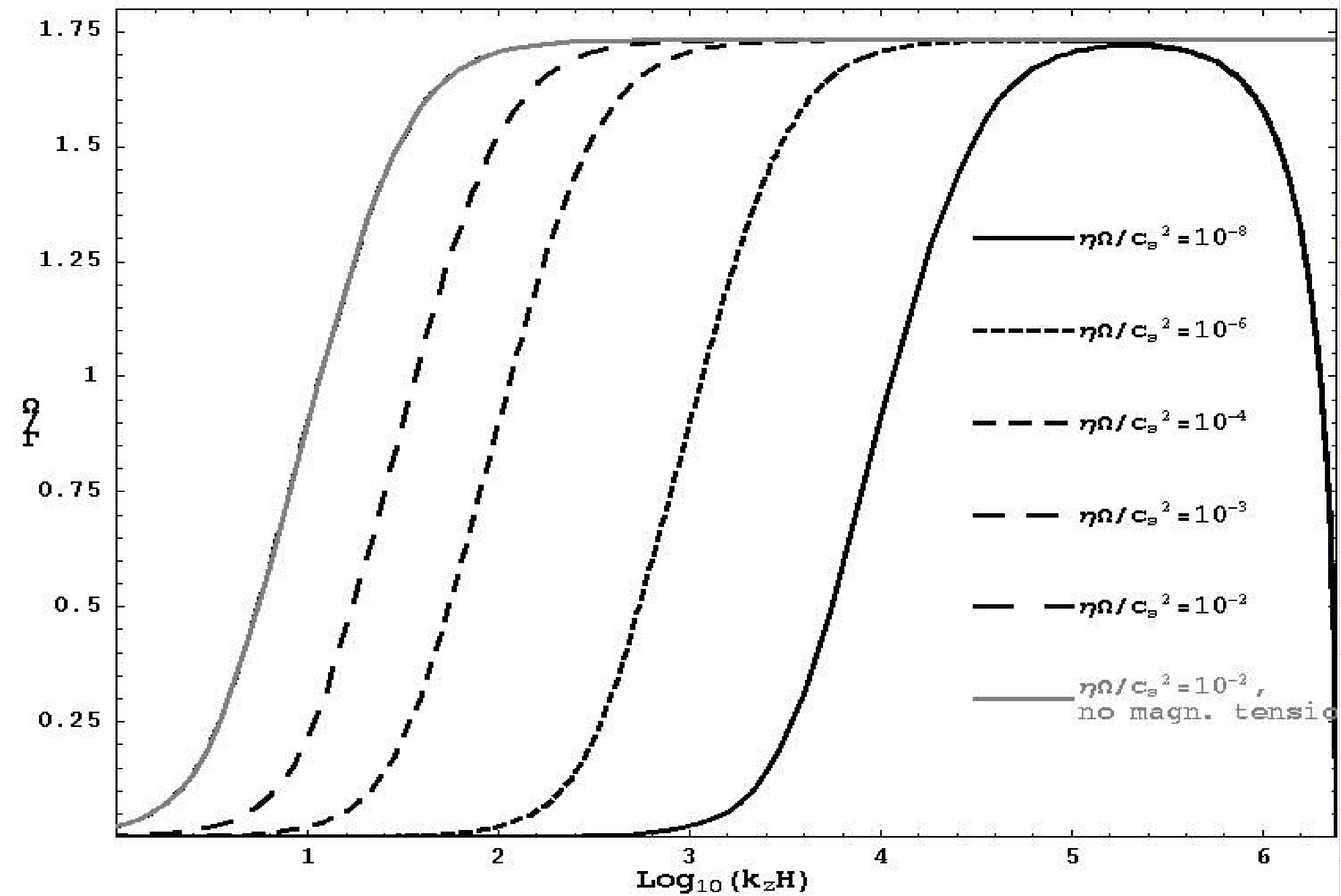
$$\hat{\eta}_\nu = \eta_\nu/\left(c_s H\right) = \eta_\nu \Omega/c_s^2$$

The MVI as Modification of the MRI

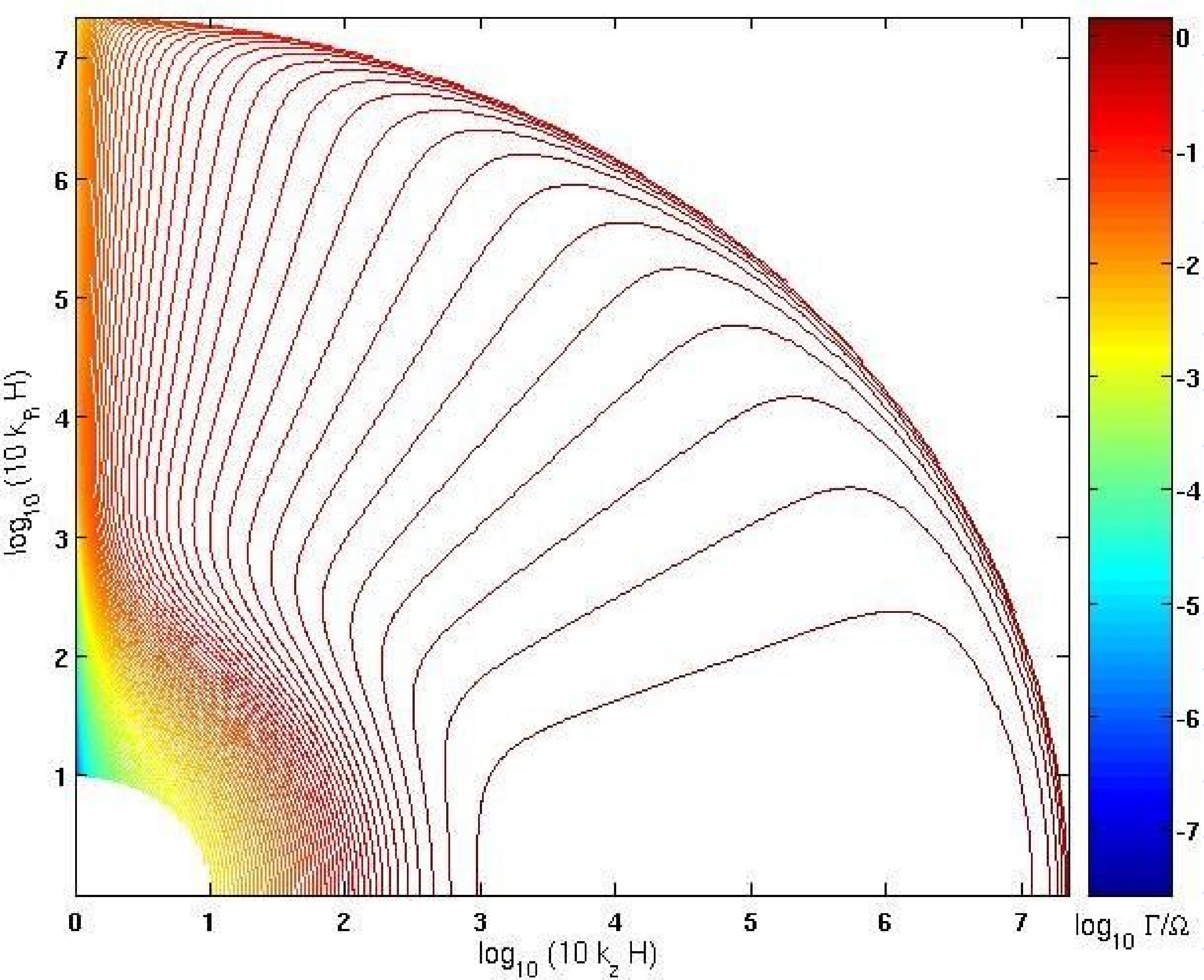


dispersion relation (growth rate as function of wavenumber) for Keplerian disk for different collisional frequencies ν_{ii}

Dispersion Relation of the MVI in More Appropriate Units

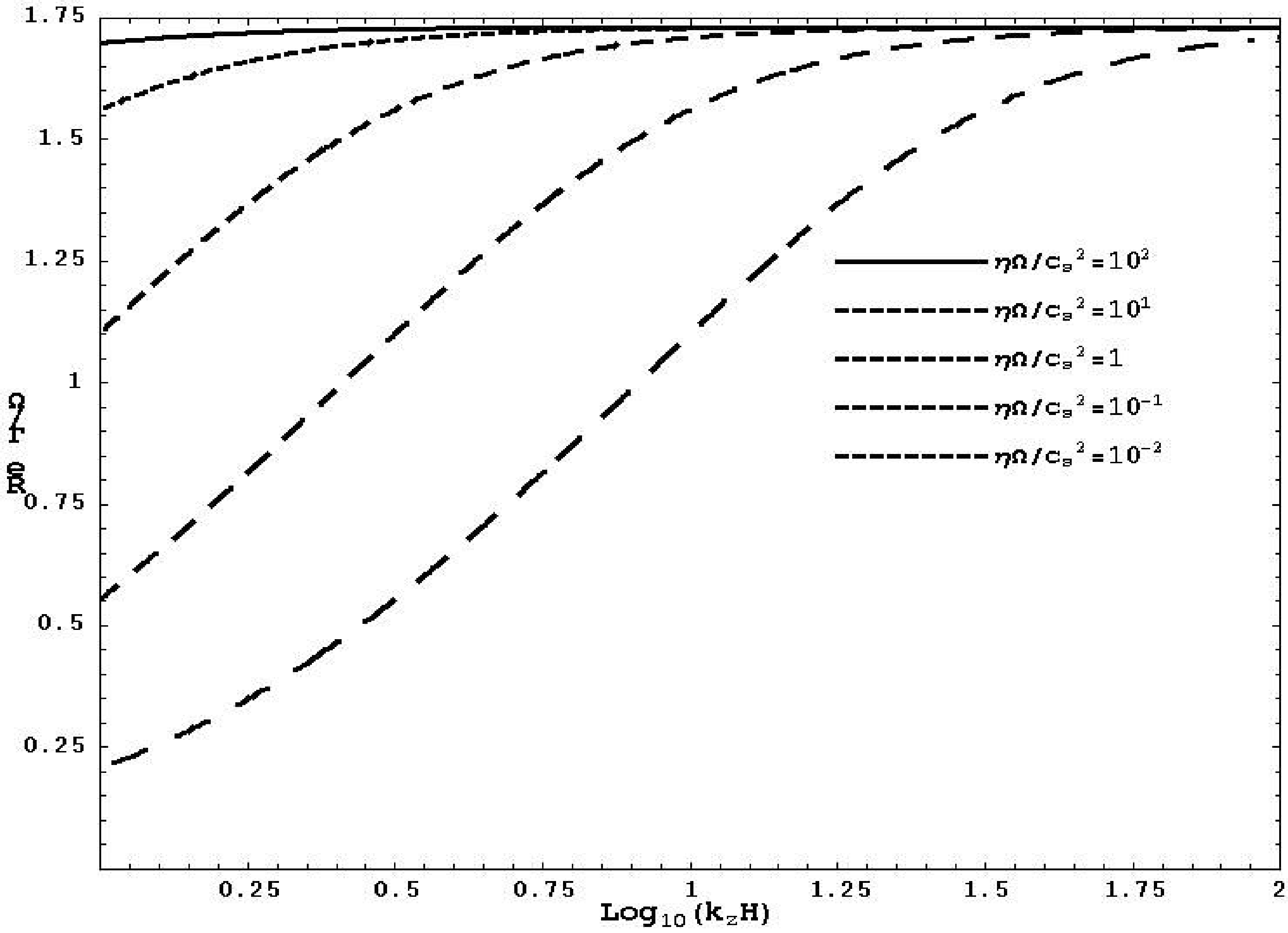


Full Axisymmetric Dispersion of MVI

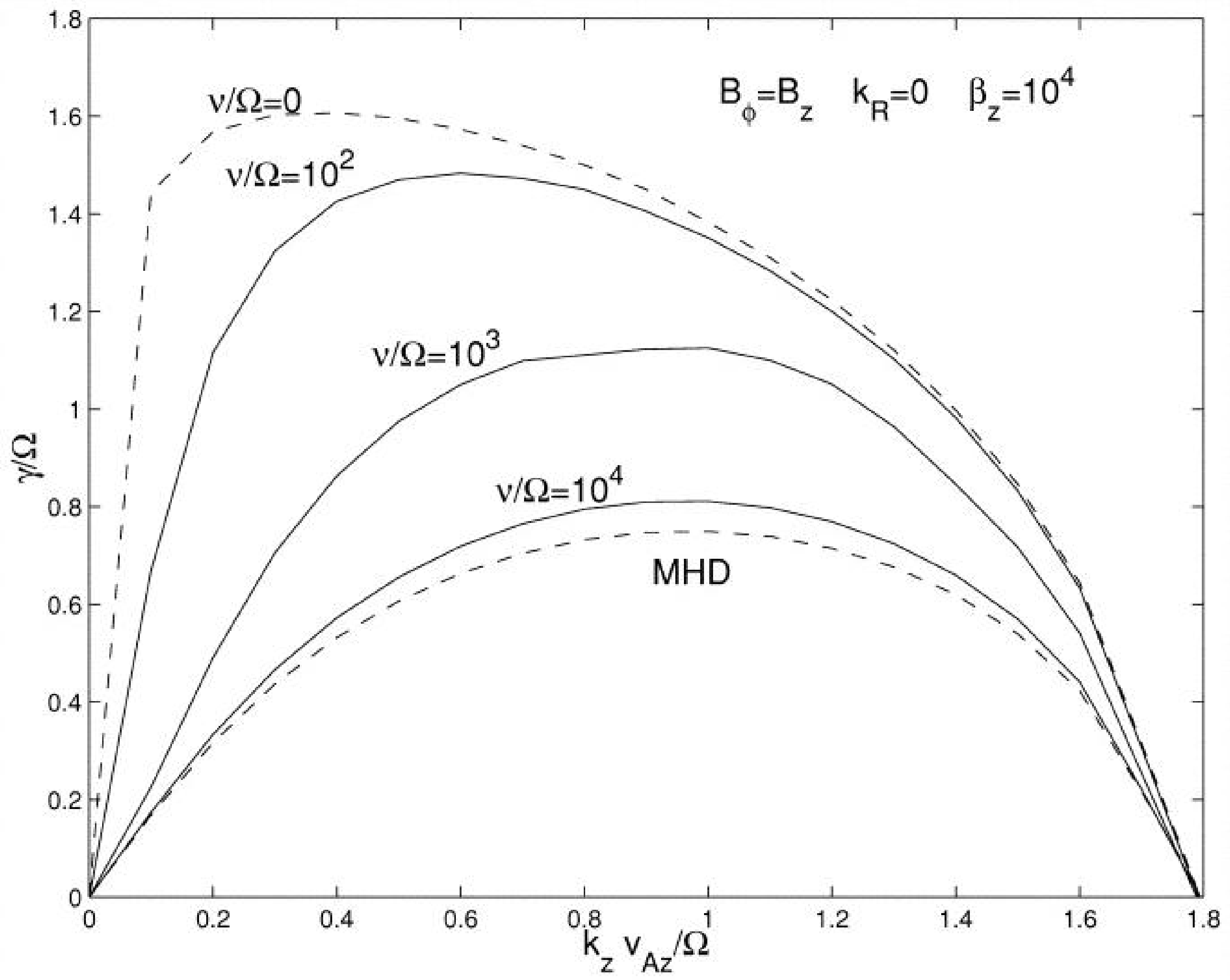


growth rate as function of wave number for $\beta=10^{12}$ plasma. Growth rate goes to zero as $k_z < k_R$.

More Careful Analysis (With Finite Compressibility) Yields Similar Dispersion Relation



Kinetic Analysis Yields Results Consistent With Fluid Approach



plot taken from E. Quataert, W. Dorlund and G. Hammett, Ap. J. **577**, 524 (2002). A kinetic analysis with finite collision frequency is done in the above article.

Directions for Further Research

- Inclusion of anisotropic finite thermal conductivity in the dilute magnetized plasma, since the viscous and ion thermal diffusion coefficients are of the same order.
- Numerical simulations in a toy model unstable to the MVI.
- Application to real astrophysical problem (RIAFs for example).