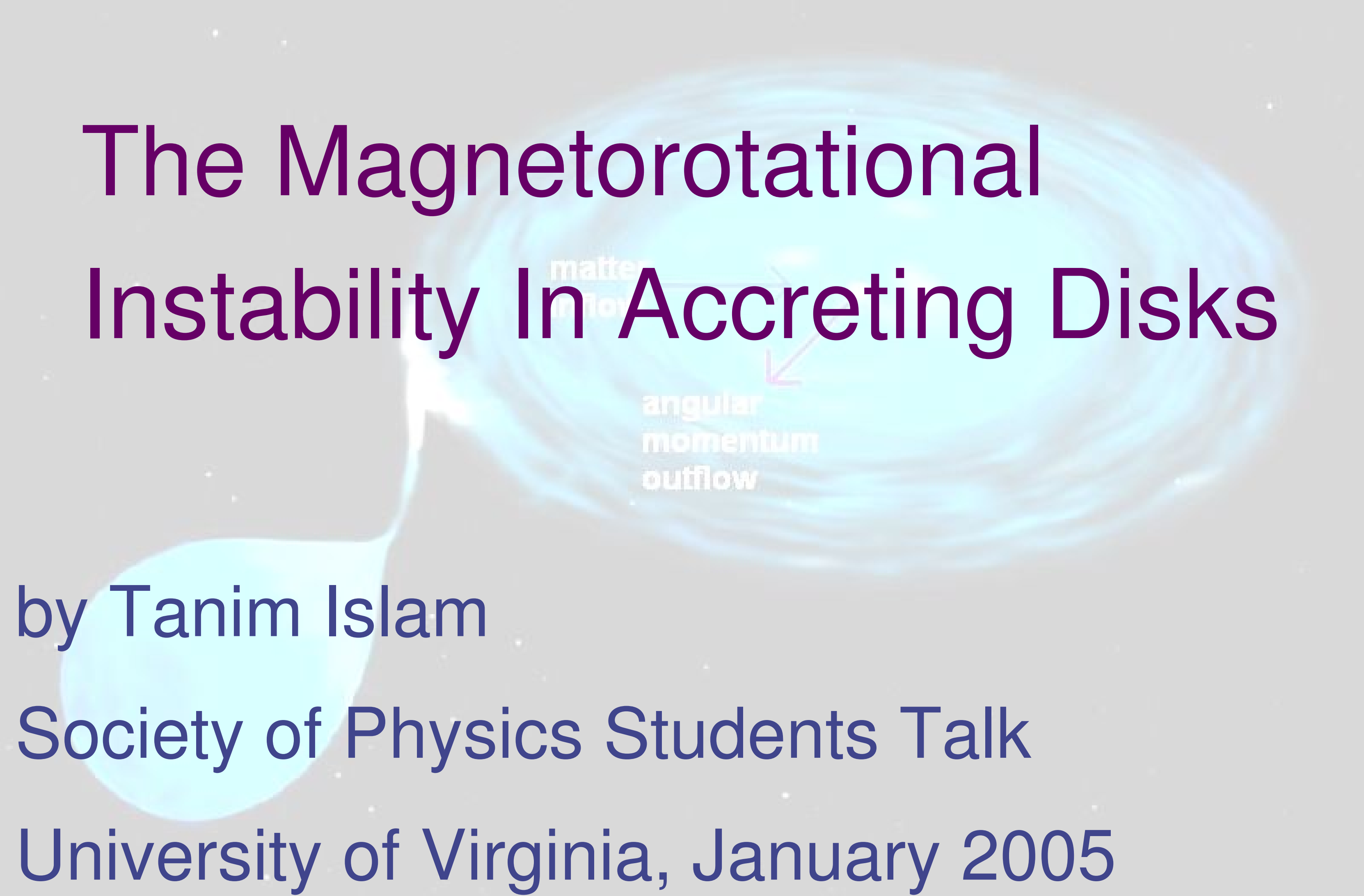
The Magnetorotational Instability In Accreting Disks



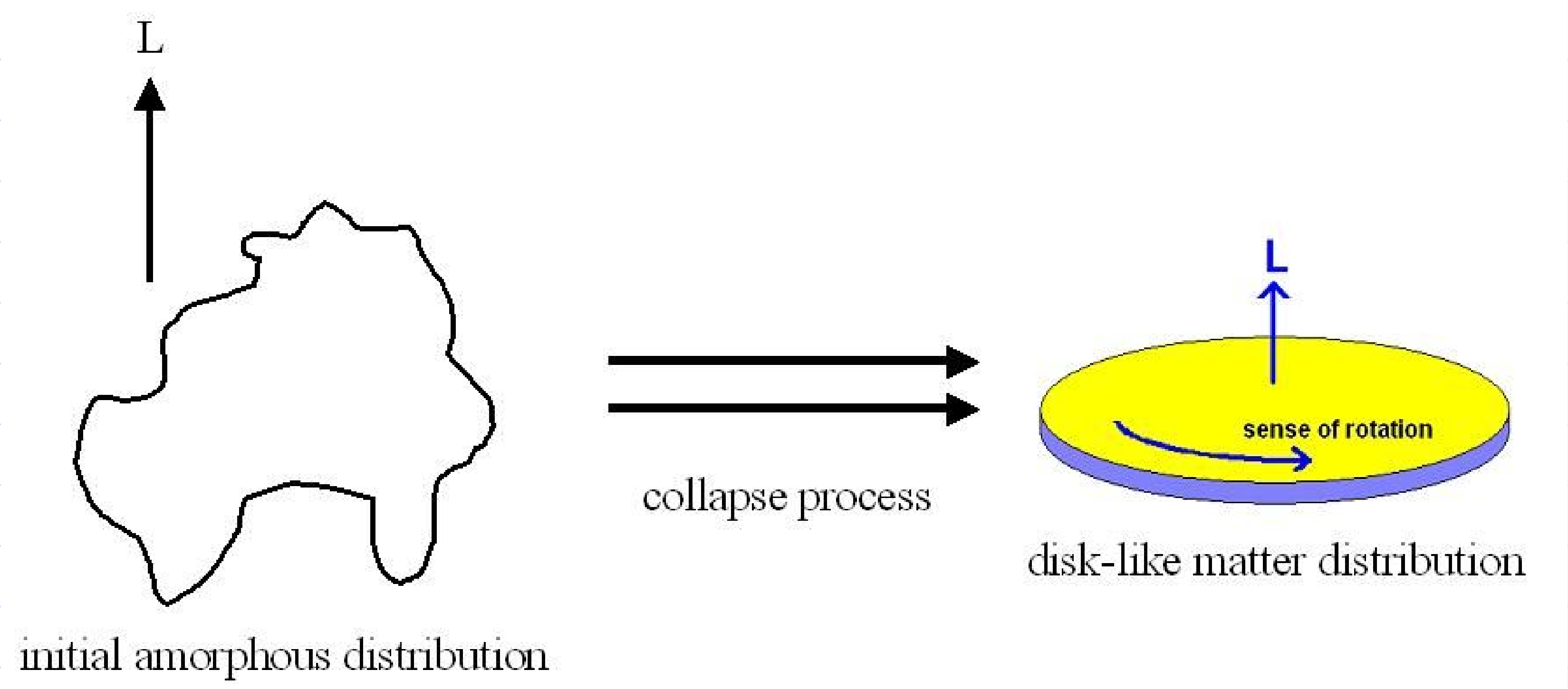
by Tanim Islam

Society of Physics Students Talk

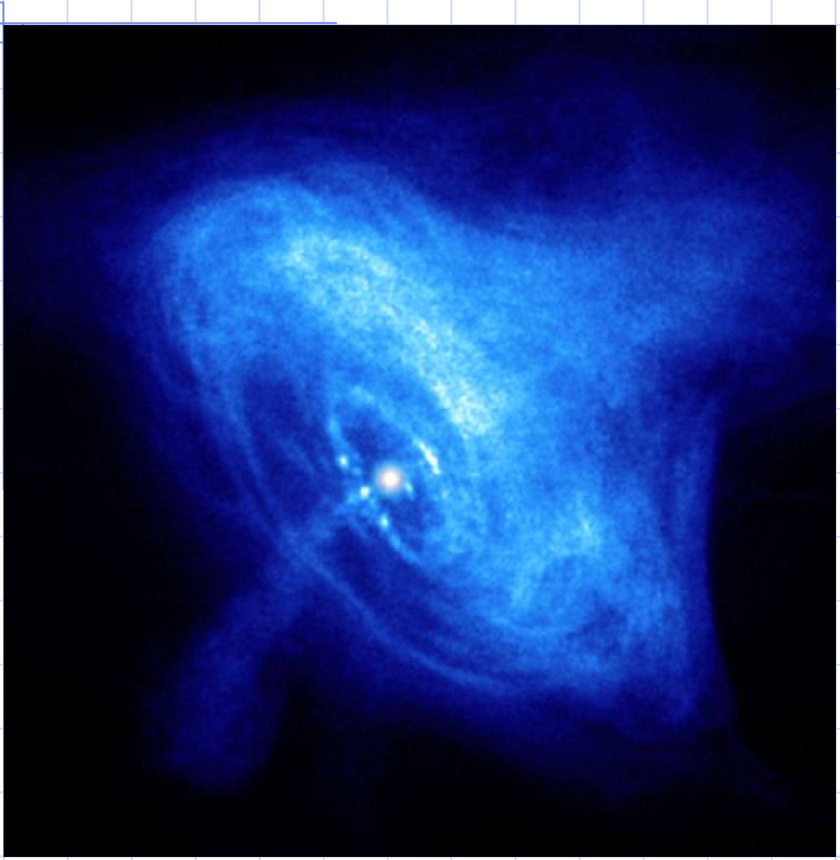
University of Virginia, January 2005

Extended objects that rotate are naturally disk-like

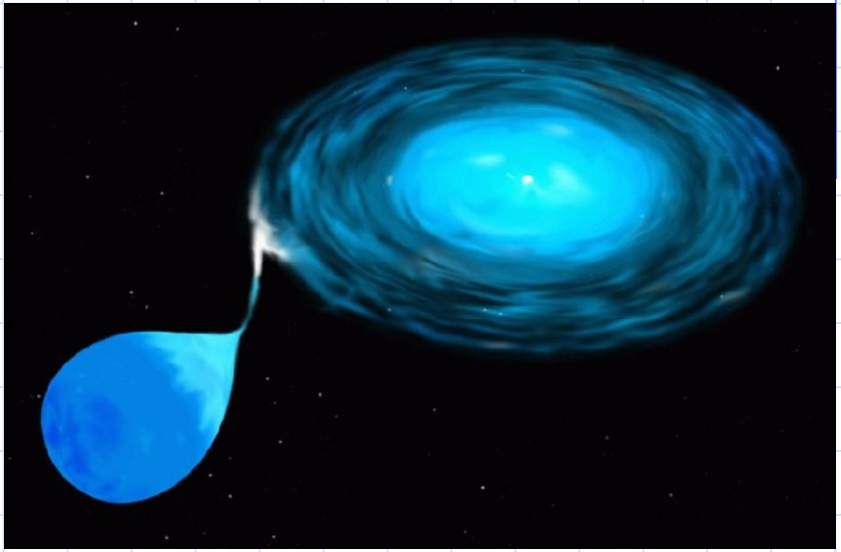
It is relatively easy to lose kinetic energy (due to friction and dissipation), but “hard” to lose angular



Accretion (the collection of matter onto a central object) is observed in a variety of objects, for example…



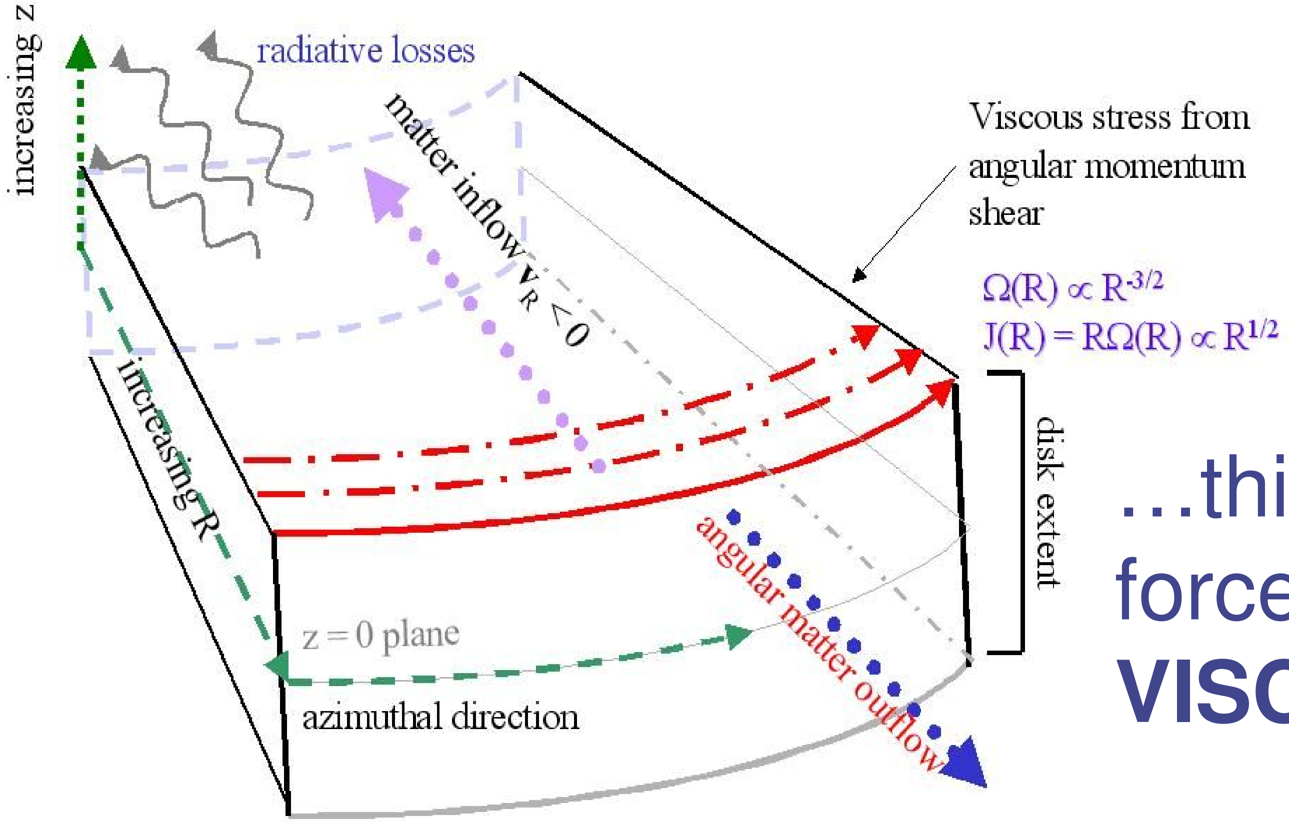
compact objects (such as pulsars, of which the above is the Crab Pulsar)



binary systems (the above is an artist’s sketch of what one looks like)

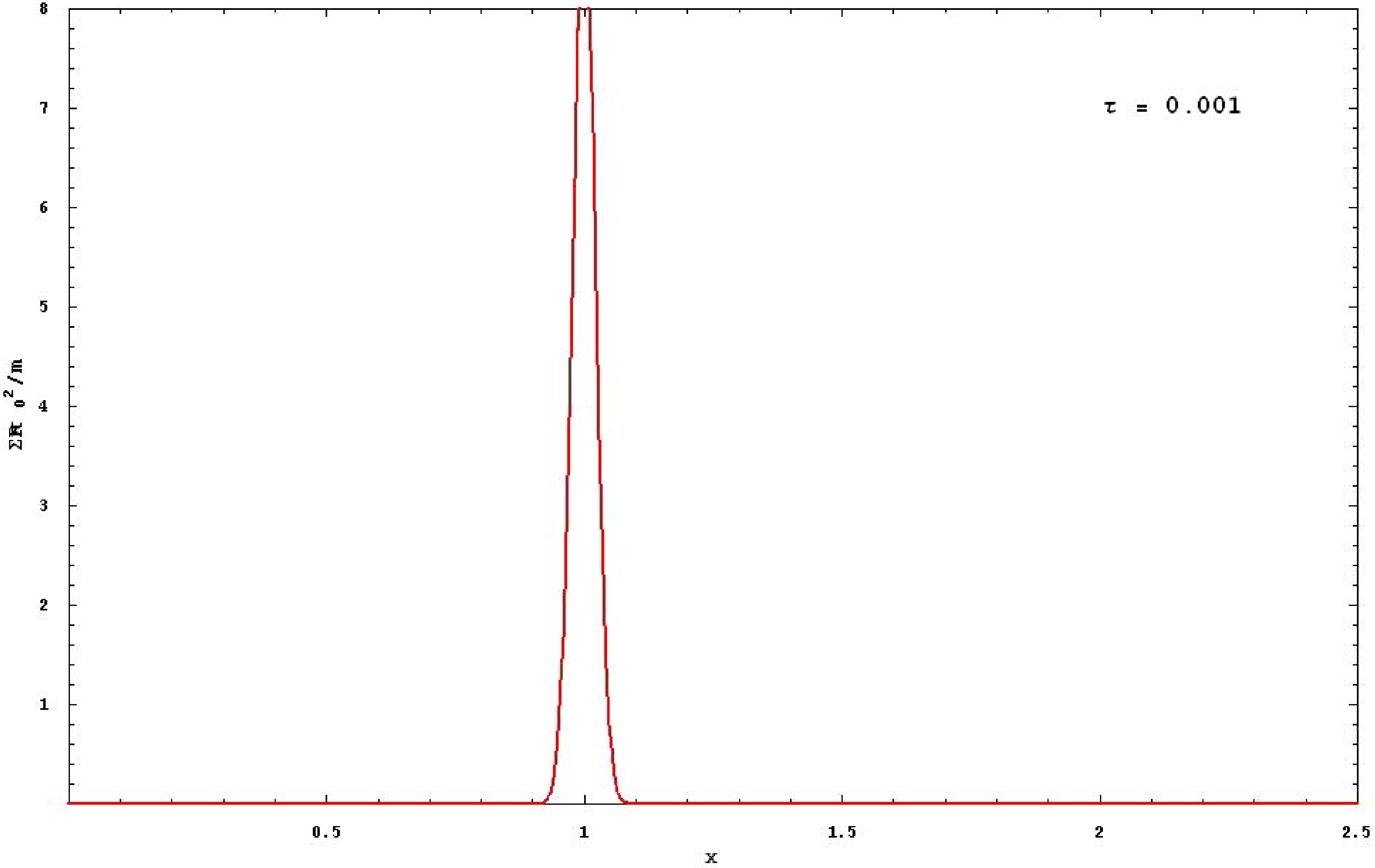
The most straightforward method to explain accretion is one in which a frictional force in the disk, BETWEEN adjacent surfaces, allows for:

- transfer of angular momentum (spin) **outwards**



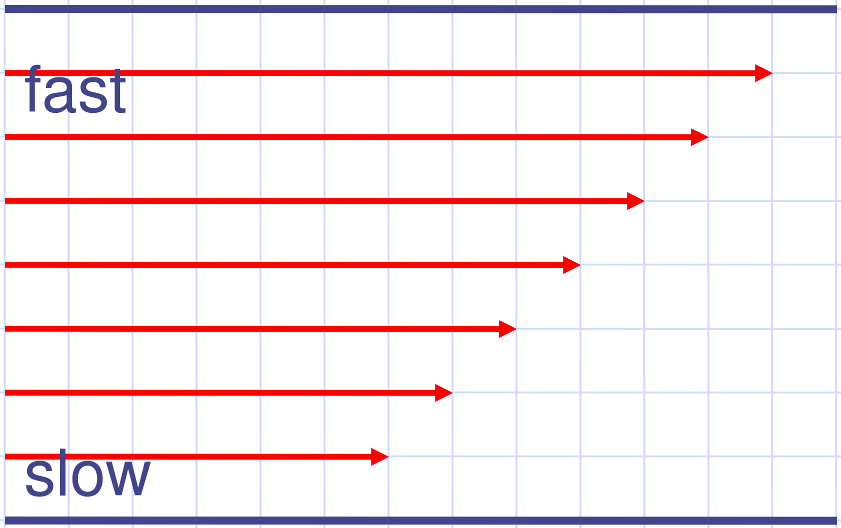
…this frictional force is called **VISCOSITY**

ANIMATION OF THE VISCOUS TRANSPORT OF MATTER AND ANGULAR MOMENTUM IN AN ACCRETION DISK



VISCOSITY: SIMPLE DEFINITION

a force of friction between fluids moving at different velocities relative to each other.



a velocity gradient in the pipe leads to “viscous forces” that tend to make the fluid move all at the same velocity

In the presence of a viscosity alone, the velocity profile in a fluid **diffuses** (is described by a diffusion coefficient):

- in Cartesian coordinates: $$\frac{\partial \mathbf{v}}{\partial t} = \eta_\nu \left(\frac{\partial^2 \mathbf{v}}{\partial x^2} + \frac{\partial^2 \mathbf{v}}{\partial y^2} + \frac{\partial^2 \mathbf{v}}{\partial z^2} \right)$$
- in general coordinates: $$\frac{\partial \mathbf{v}}{\partial t} = \eta_\nu \nabla^2 \mathbf{v}$$

Where η_ν is the diffusion coefficient

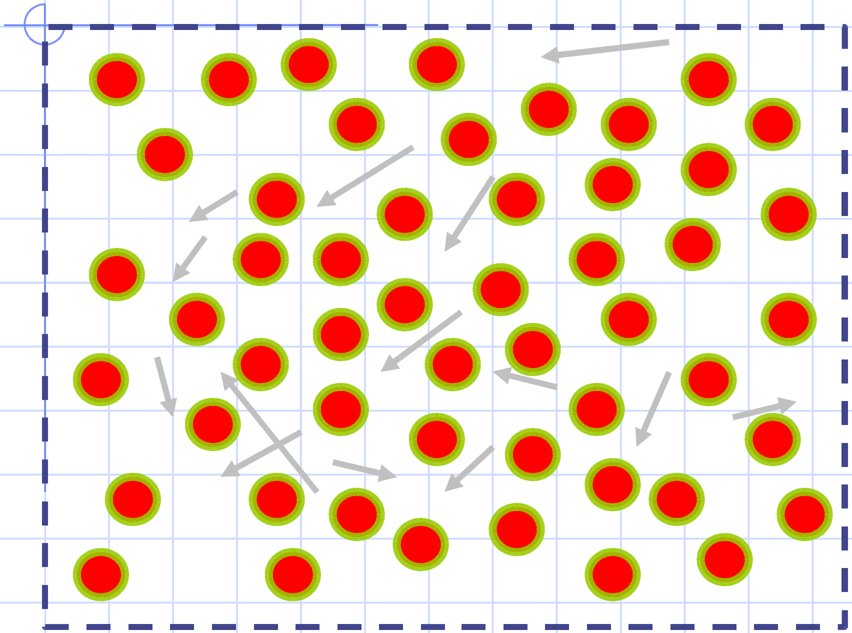
η_ν ~ (collision frequency) x (path length)^2

~ (particle velocity)^2 x (collision timescale)

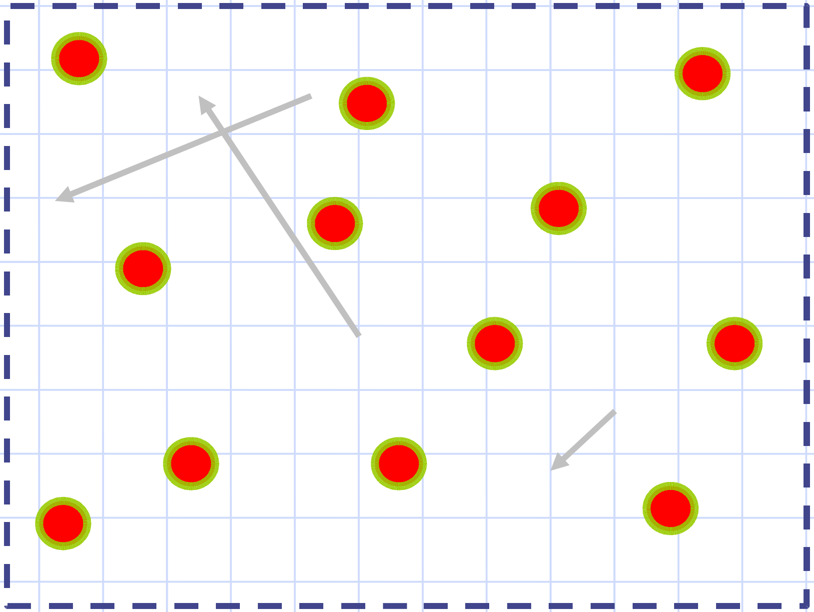
similar equations describe the diffusion of **heat** (thermal energy) and **particles**

ANIMATION OF DIFFUSION

Particle Model of Viscosity



high particle density, short collision times, **low viscosity**



low particle density, long collision times, **high viscosity**

Viscosity of Everyday Fluids

Syrup: η_ν (SI) = 3×10^{-2} m^2 s^{-1}

Water: η_ν (SI) = 10^{-5} m^2 s^{-1}

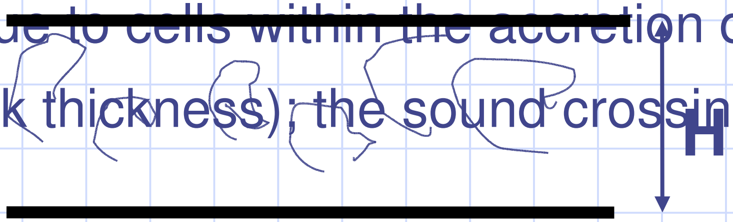
Air: η_ν (SI) = 10^{-5} m^2 s^{-1}

Astrophysical Problem

- The density and temperature of typical accretion disks yields a viscosity **too low** and a disk size **too high** – disc accretion due to collisional diffusion is **TOO SLOW**!!
  - Ex: For a typical 1 solar mass compact object or star, with an accretion disk size of 1 AU, the time scale for diffusive accretion: **100 billion years** – nothing ever forms!
  - Typical accretion disks, with masses of 0.1 solar masses or less, either (1) hardly accrete, or (2) require masses orders of magnitude larger than the central object (not seen)
- Observations of light curves of cataclysmic variables seem to imply that diffusion over a typical sized disk takes place over **hours to weeks**.
- Some “diffusive” process must occur within the disk – the viscosity must be orders of magnitude higher than what could be explained with collisional processes.

α Viscosity Paradigm

- Shakura and Sunyaev* from dimensional arguments, and based on what was believed to be the disk properties characterizing viscosity, believed diffusion was enhanced by ***hydrodynamic turbulence***.
- The turbulence is due to cells within the accretion disk. The size of the cells is **H** (disk thickness); the sound crossing speed is $\mathbf{c_s}$ (sound speed)



$$\eta_\nu = \alpha c_s H$$

where α is a constant

- A variety of different astrophysical accretion phenomena have been fit to this α viscosity model, with α<1 (but not too small)
- **PROBLEM: no self-consistent method to derive a turbulent**

*Astronomy and Astrophysics 24, 337-355 (1973)

The Magnetorotational Instability

- First discovered by Velikhov[1] and Chandrasekhar[2], and used as an explanation for rigid-body (constant Ω) rotation in stars.
- Systems in which the **angular velocity** Ω rather than angular momentum ΩR^2 (in hydrodynamic flows) are unstable to these modes.
- Instability grows at the rate of Ω at wavelengths much **smaller** than the disk height ("turbulence" within the disk arising from magnetic fields)

[1] Sov. Phys. JETP **36**, 995 (1959).

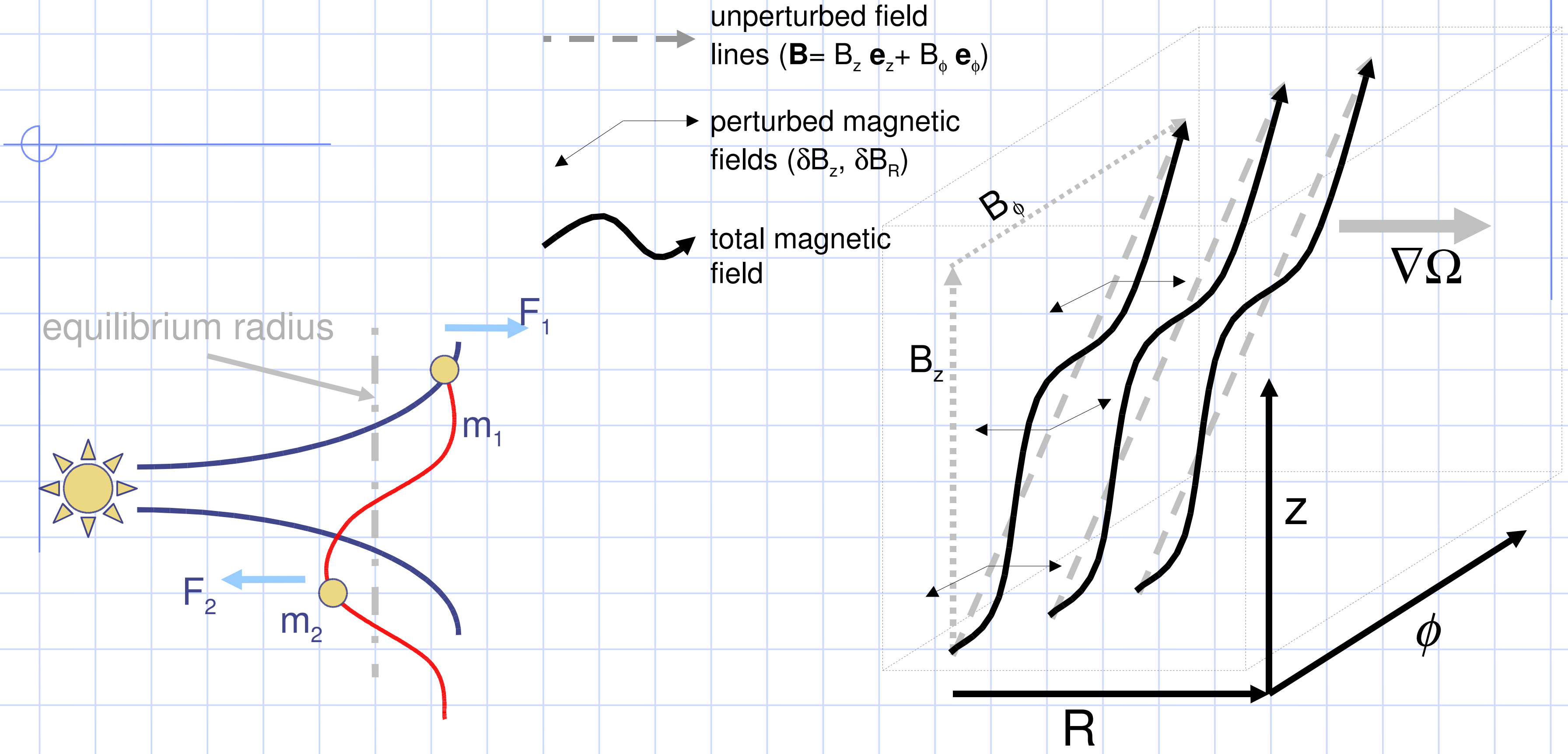
[2] Proc. Nat. Acad. Sci. USA **46**, 53 (1960).

Astrophysical Application

- Balbus and Hawley* showed that the MRI could be applied under much more general and universal conditions (namely that Ω decreases outward radially) and is a global instability (important wherever in the disk that the above condition is met).
- First to apply the use of the MRI in explaining magnetized turbulence, hence enhanced viscosity, within accretion disks.
- From 2D and 3D simulations, showed that magnetic fields from even a weak level saturate at pressures comparable to the gas pressure.
- Determined a ~ 1 (or not much smaller), as Shakura and Sunyaev first proposed!

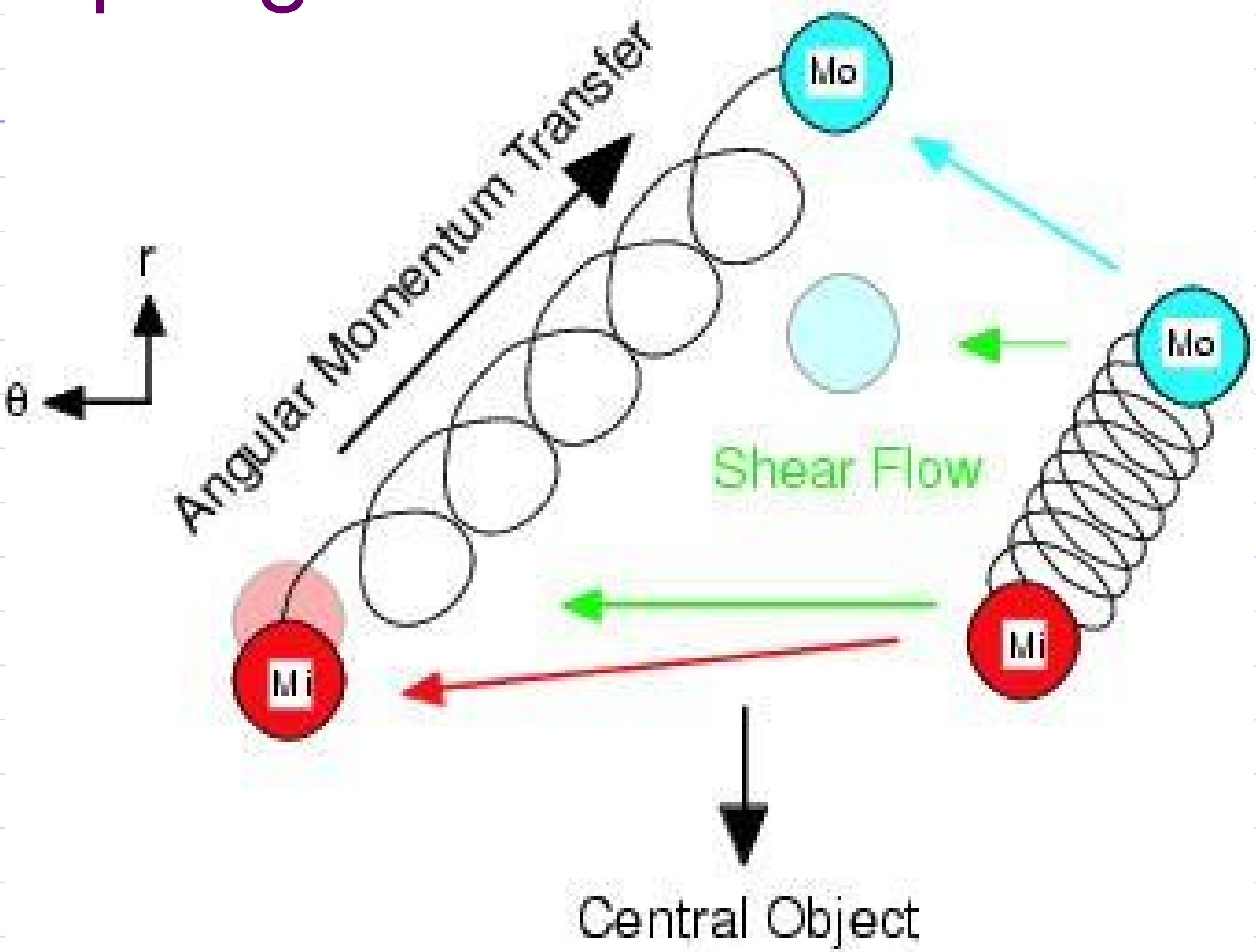
*Astrophys. Jour. 376, 214 (1991)

Schematic Model of MRI



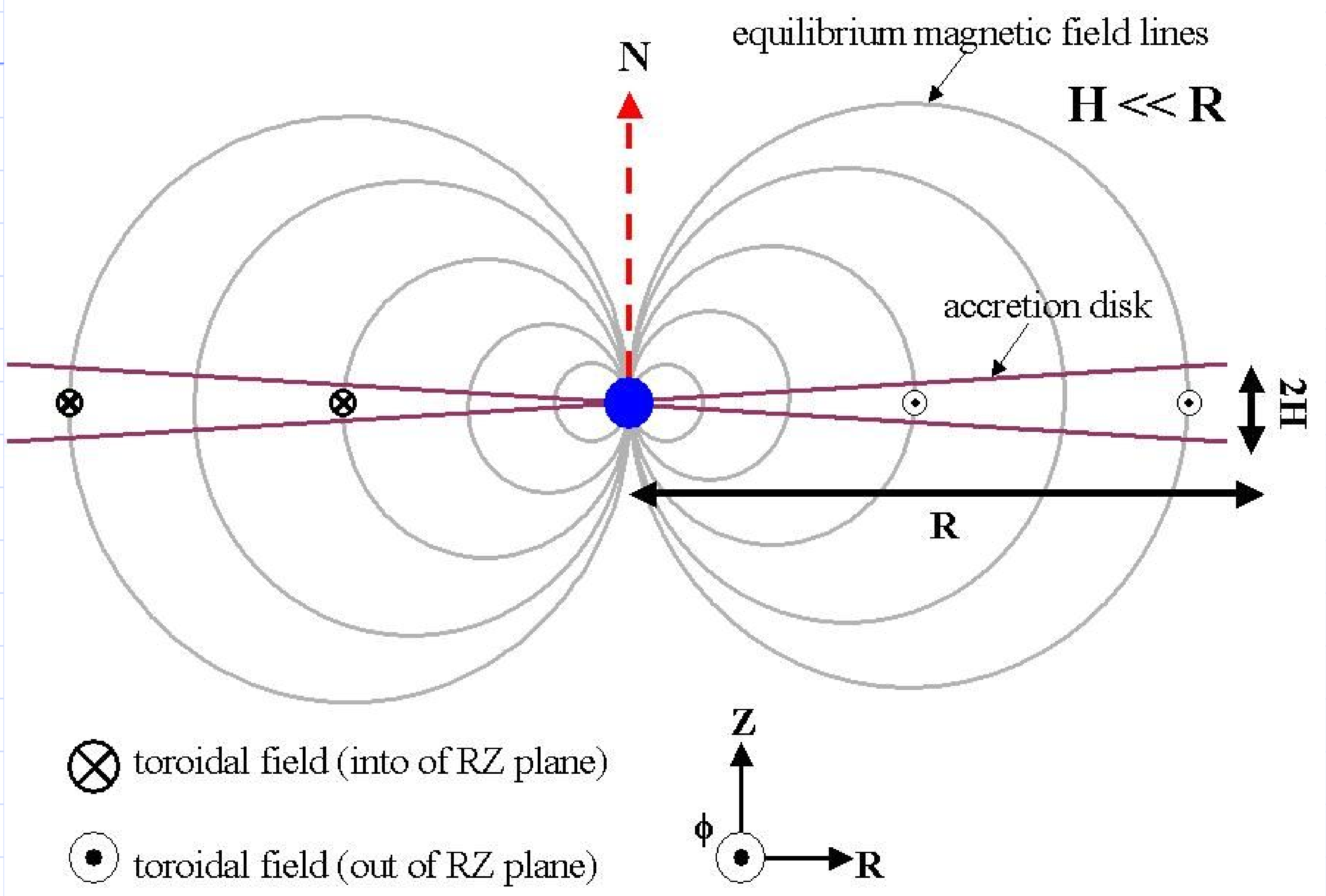
- Points on a magnetic field line are forced to corotate (same Ω)
- The points further out from the equilibrium tend to accelerate outward, while points inside accelerate inwards
- This is all quenched at small enough wavelengths due to the effects of magnetic tension

Spring Model of the MRI

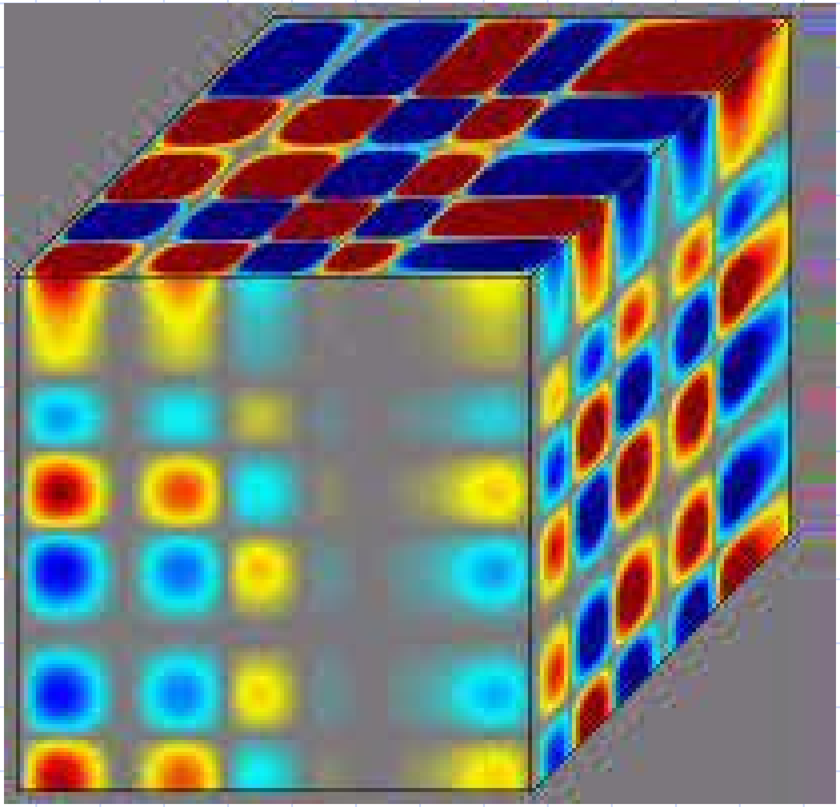


Koichi Noguchi 2002

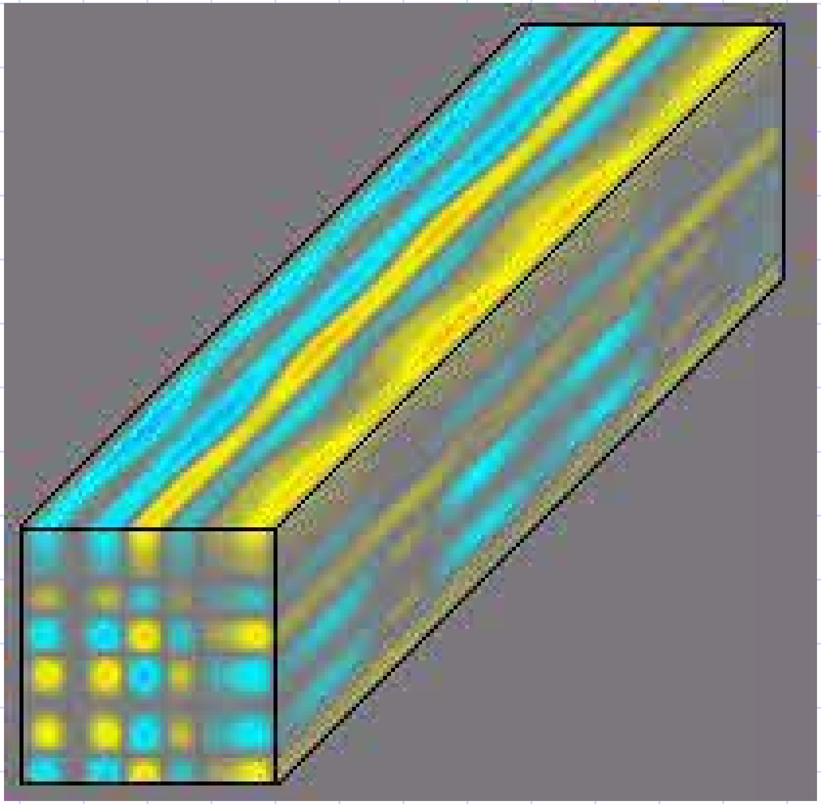
The MRI in accretion disks



Nonlinear Simulations I



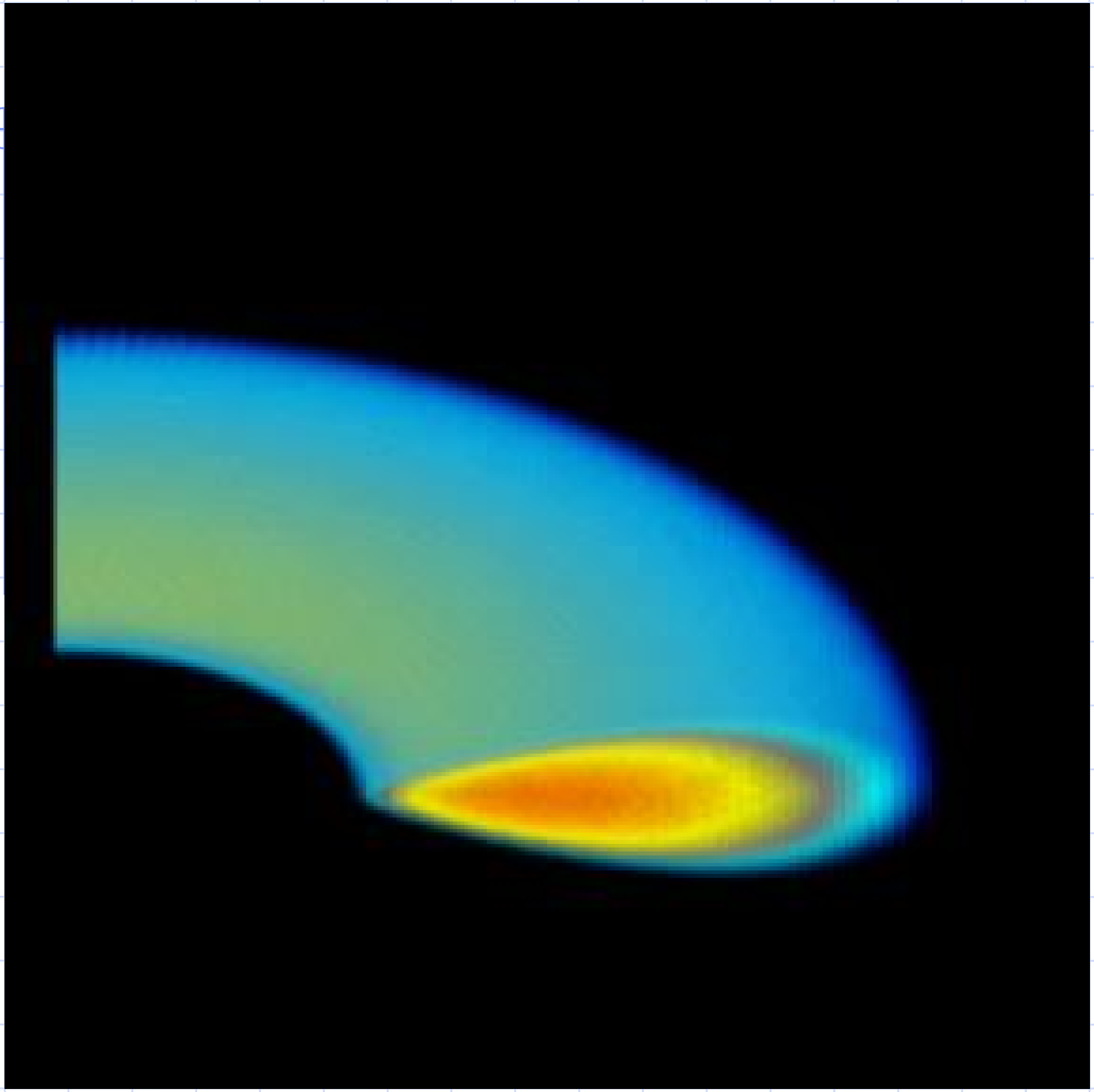
no magnetic fields



with magnetic fields

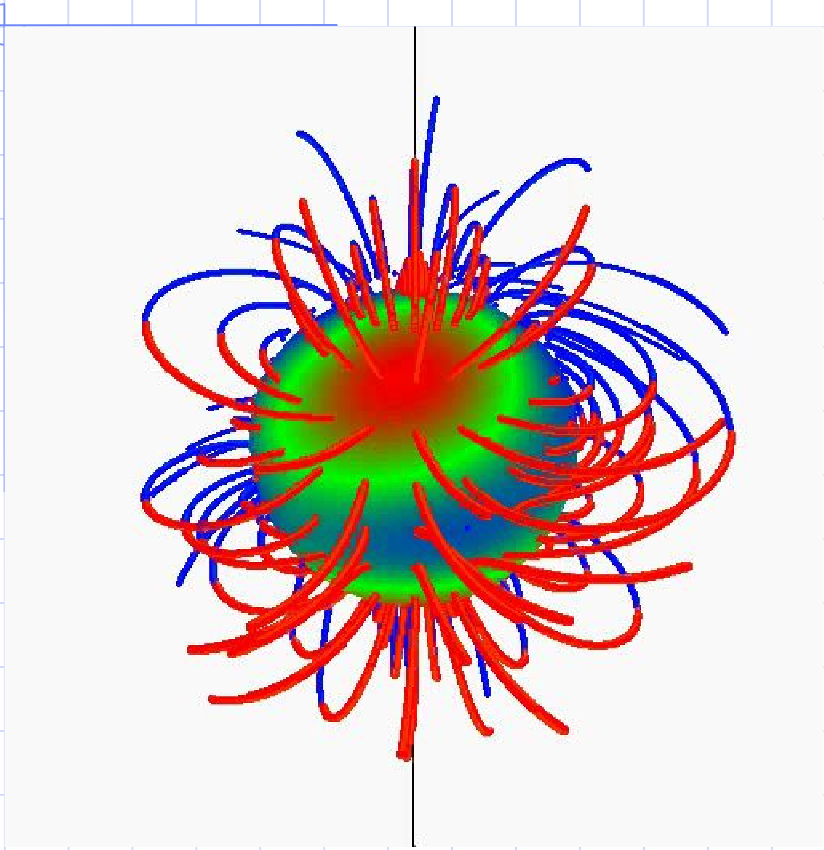
Taken from http://www.astro.virginia.edu/VITA/accdisk.html

Nonlinear Simulations II

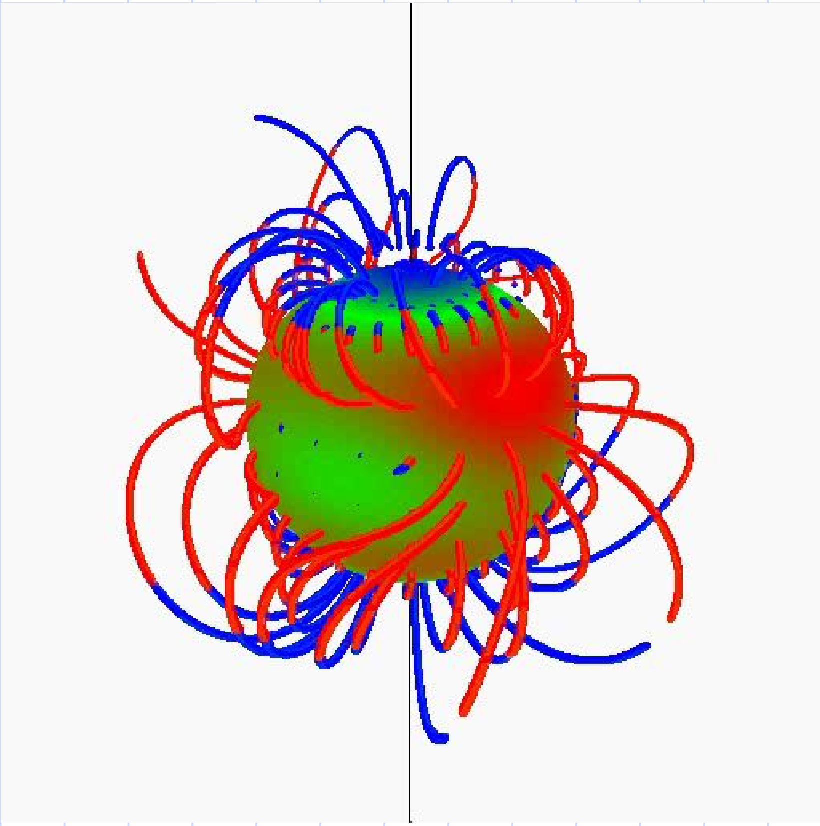


Taken from http://www.astro.virginia.edu/VITA/papers/plunge

Simulation of MRI in the laboratory



m=1



m=0

Taken from http://complex.umd.edu/mri/

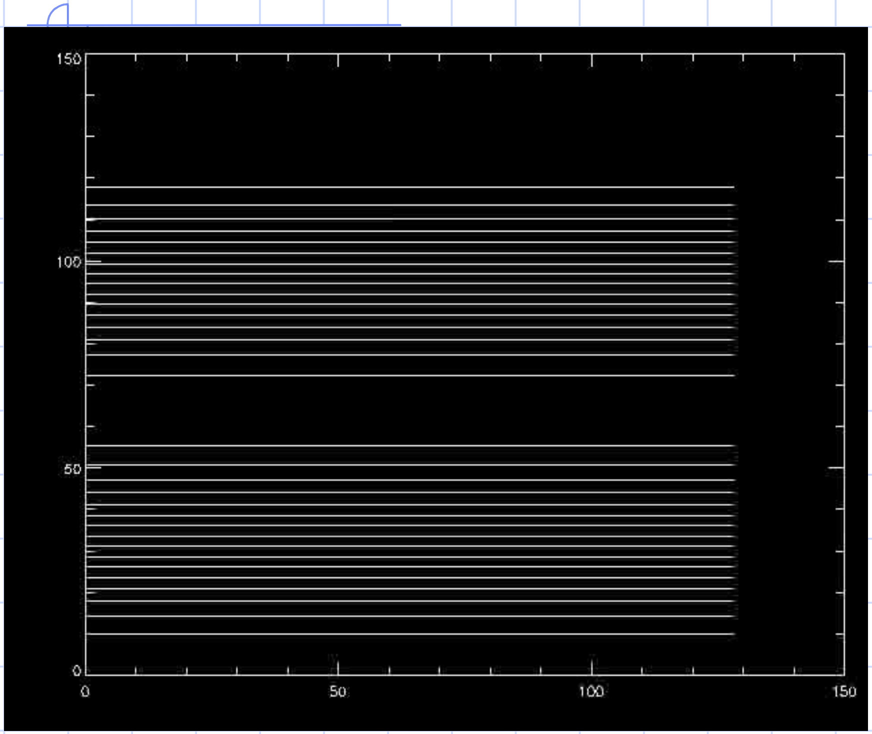
The MRI is simply one manifestation of how magnetic fields modify disk stability

Instability Conditions

	nonmagnetized	magnetized
angular momentum transfer	angular momentum ($\Omega^2 R$) decreases radially outwards	angular velocity (Ω) decreases radially outward or upward
energy transfer	entropy density (heat content) decreases upward or radially outward.	temperature **T** decreases upward or radially outward – outer or upper regions cooler.

Taken from S. Balbus, Astroph. Jour. **562**, 909 (2001).

Nonlinear Thermal Instability In Magnetized Plasma



magnetic field lines

temperature

Taken from http://www.astro.princeton.edu/~iparrish