The Magnetorotational Instability In Accreting Disks

by Tanim Islam

IAP Student Talk

June 2005

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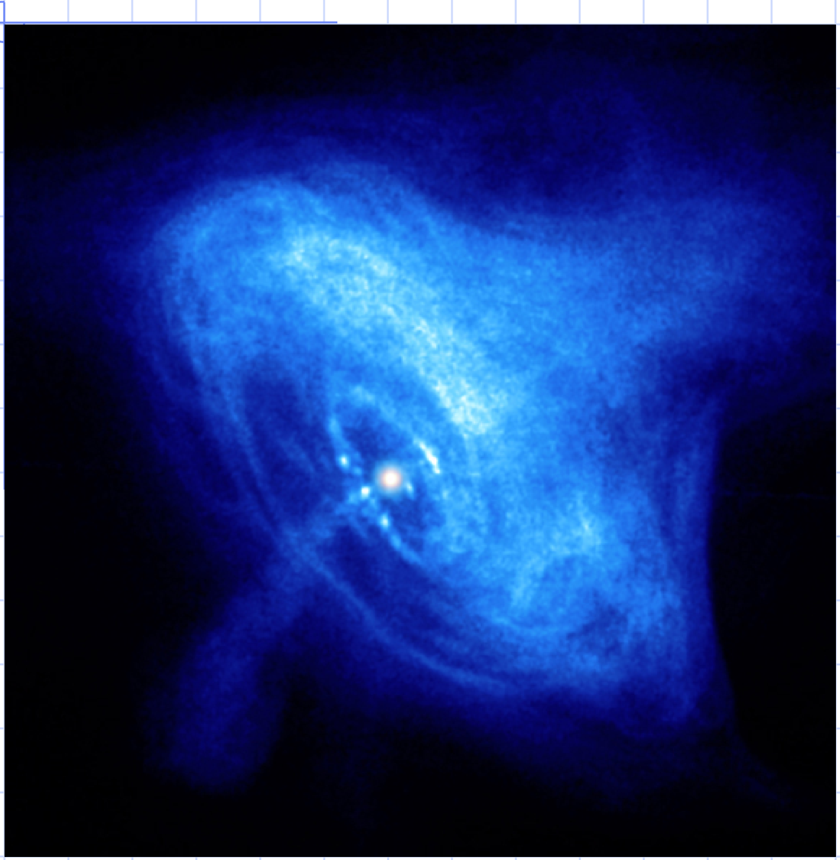

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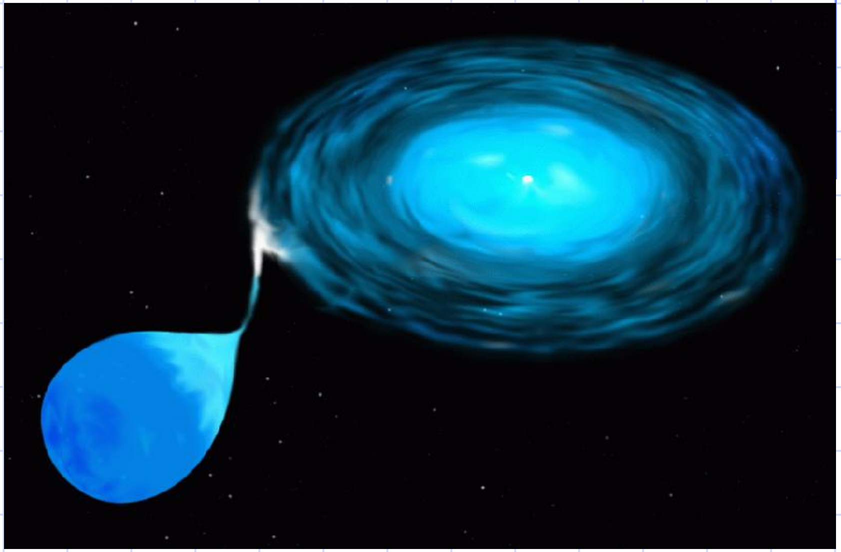

binary systems (the above is an artist’s sketch of what one looks like)

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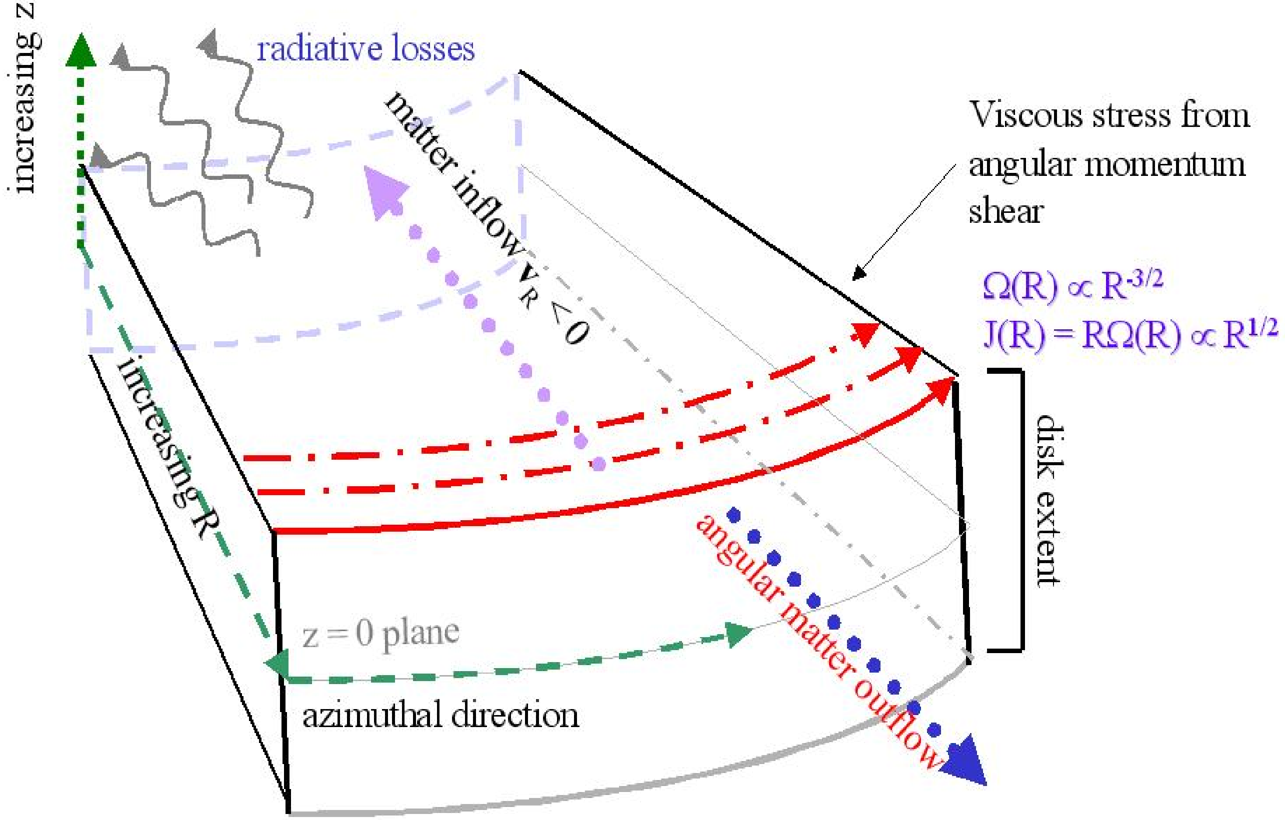

Example: Diffusion Equation For Accretion

$$\begin{aligned}
v_R(R,t) &= \frac{1}{R\Sigma d\left(R^2\Omega\right)/dR}\frac{\partial}{\partial R}\left(\eta_\nu \Sigma R^3 \frac{d\Omega}{dR}\right) \\
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- Above diffusion equation is applied to flows within thin accretion disks, but paradigm is universal in disk accretion models.
- η_ν is a phenomenological α viscosity.

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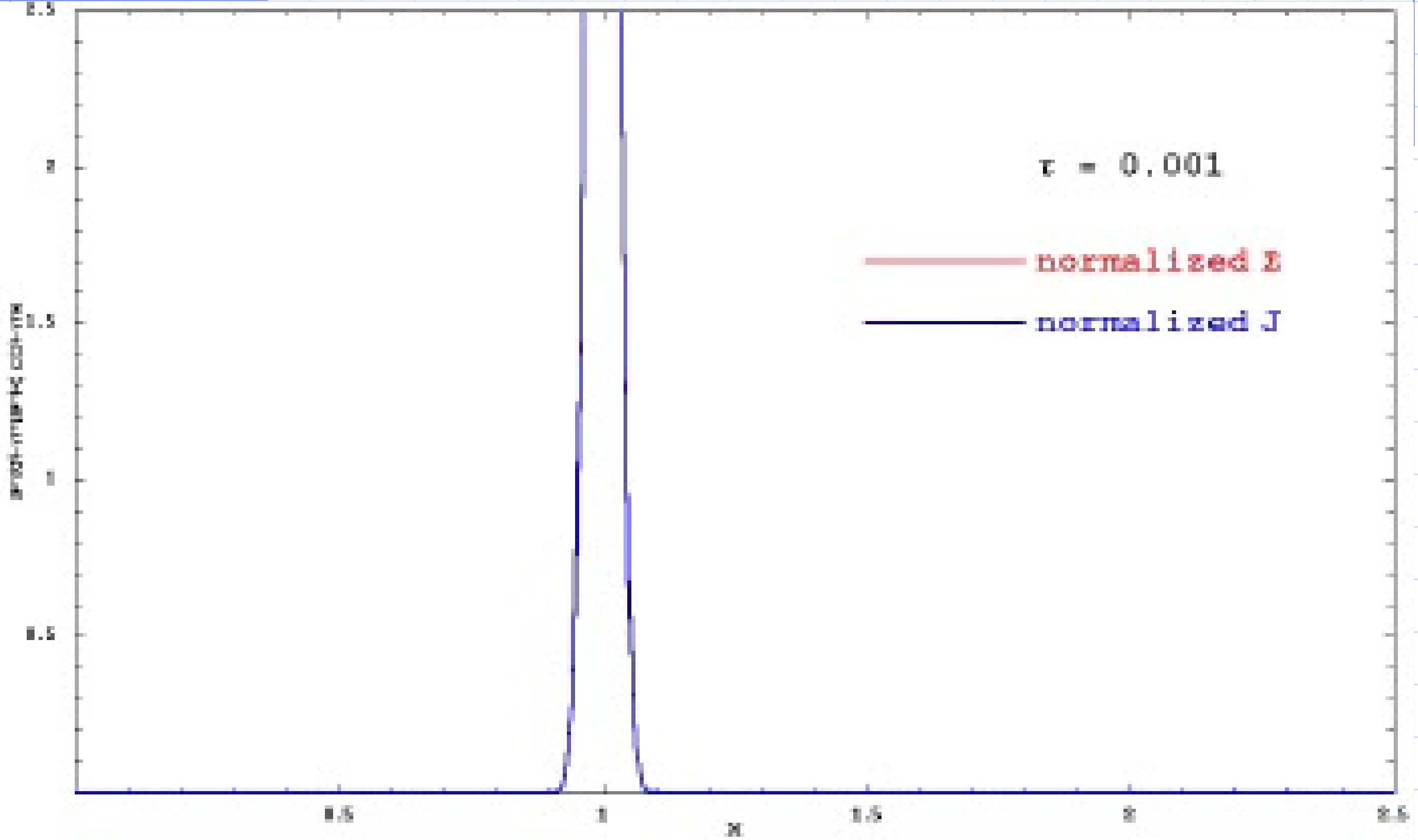

α Viscosity Paradigm

Shakura and Sunyaev [1] believed diffusion was enhanced by ***hydrodynamic turbulence*** – the size of the cells is **H** (disk thickness); the sound crossing speed is $\mathbf{c_s}$ (sound speed)

$$\eta_\nu = \alpha c_s H \gg c_s^2/\nu_{ii}$$

α	dimensionless parameter
ν_{ii}	ion-ion collision rate
c_s^2/ν_{ii}	order-of-magnitude estimate of the viscosity, resulting in accretion timescales of order $10^{10} - 10^{12}$ years

The Magnetorotational Instability

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- Systems in which the **angular velocity** Ω rather than angular momentum ΩR^2 (in hydrodynamic flows) are unstable to these modes.
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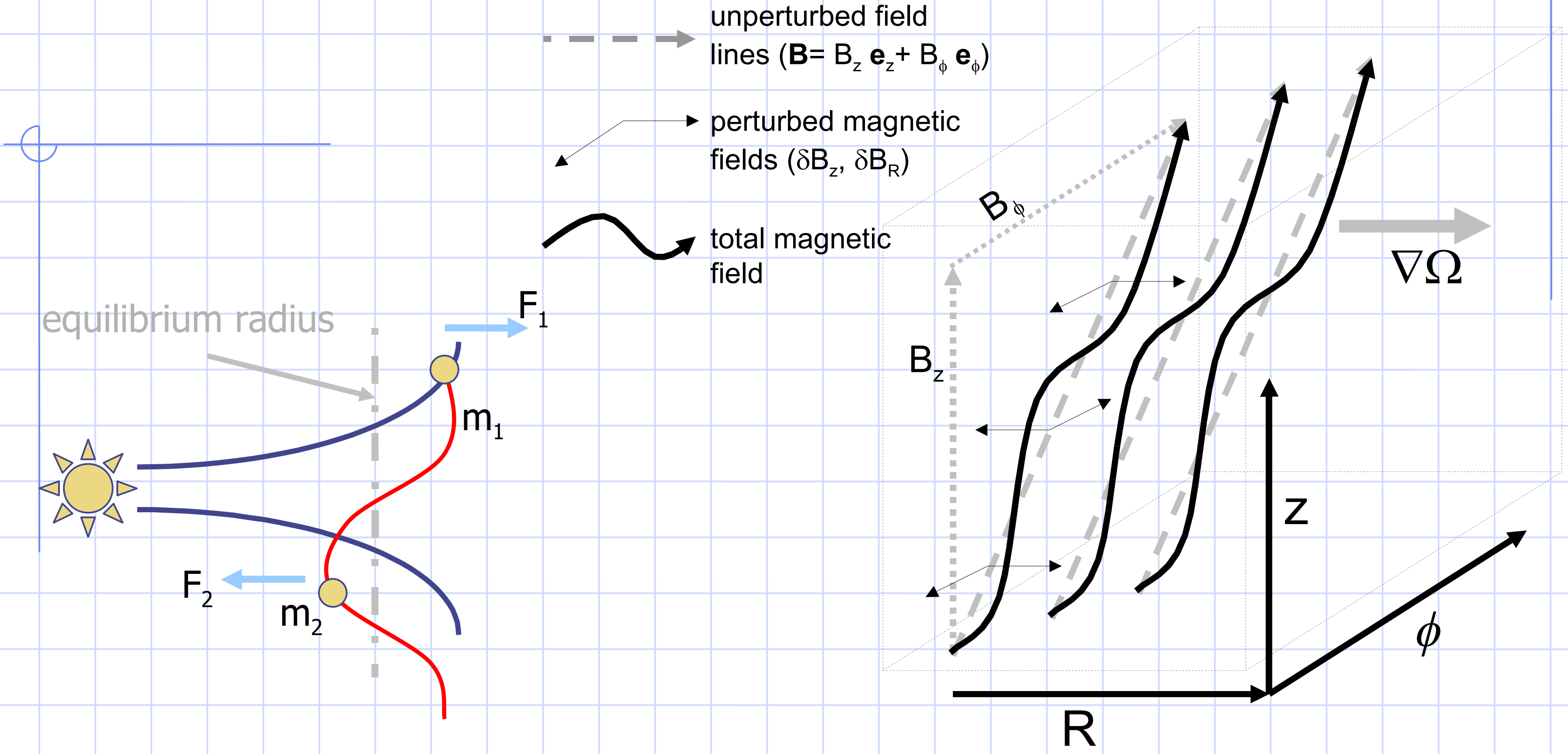
Schematic Model of MRI



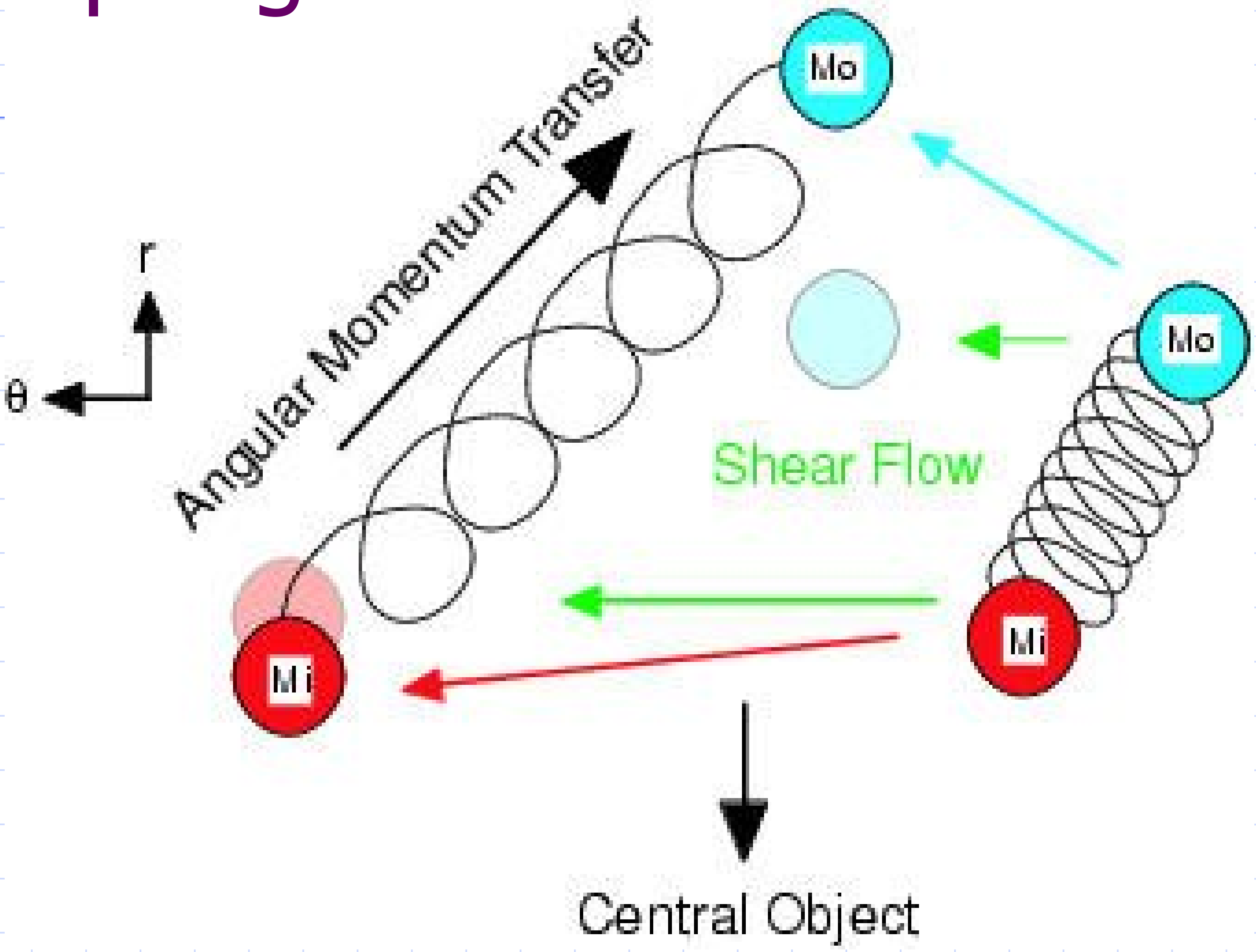
unperturbed field lines ($\mathbf{B}= B_z\,\mathbf{e}_z + B_\phi\,\mathbf{e}_\phi$)

perturbed magnetic fields (δB_z, δB_R)

total magnetic field

- Points on a magnetic field line are forced to corotate (same Ω)
- The points further out from the equilibrium tend to accelerate outward, while points inside accelerate inwards
- This is all quenched at small enough wavelengths due to the effects of magnetic tension

Spring Model of the MRI



Koichi Noguchi 2002

Astrophysical Application

- Balbus and Hawley* showed that the MRI could be applied under much more general and universal conditions (namely that Ω decreases outward radially) and is a global instability (important wherever in the disk that the above condition is met).
- First to apply the use of the MRI in explaining magnetized turbulence, hence enhanced viscosity, within accretion disks.
- From 2D and 3D simulations, showed that magnetic fields from even a weak level saturate at pressures comparable to the gas pressure.
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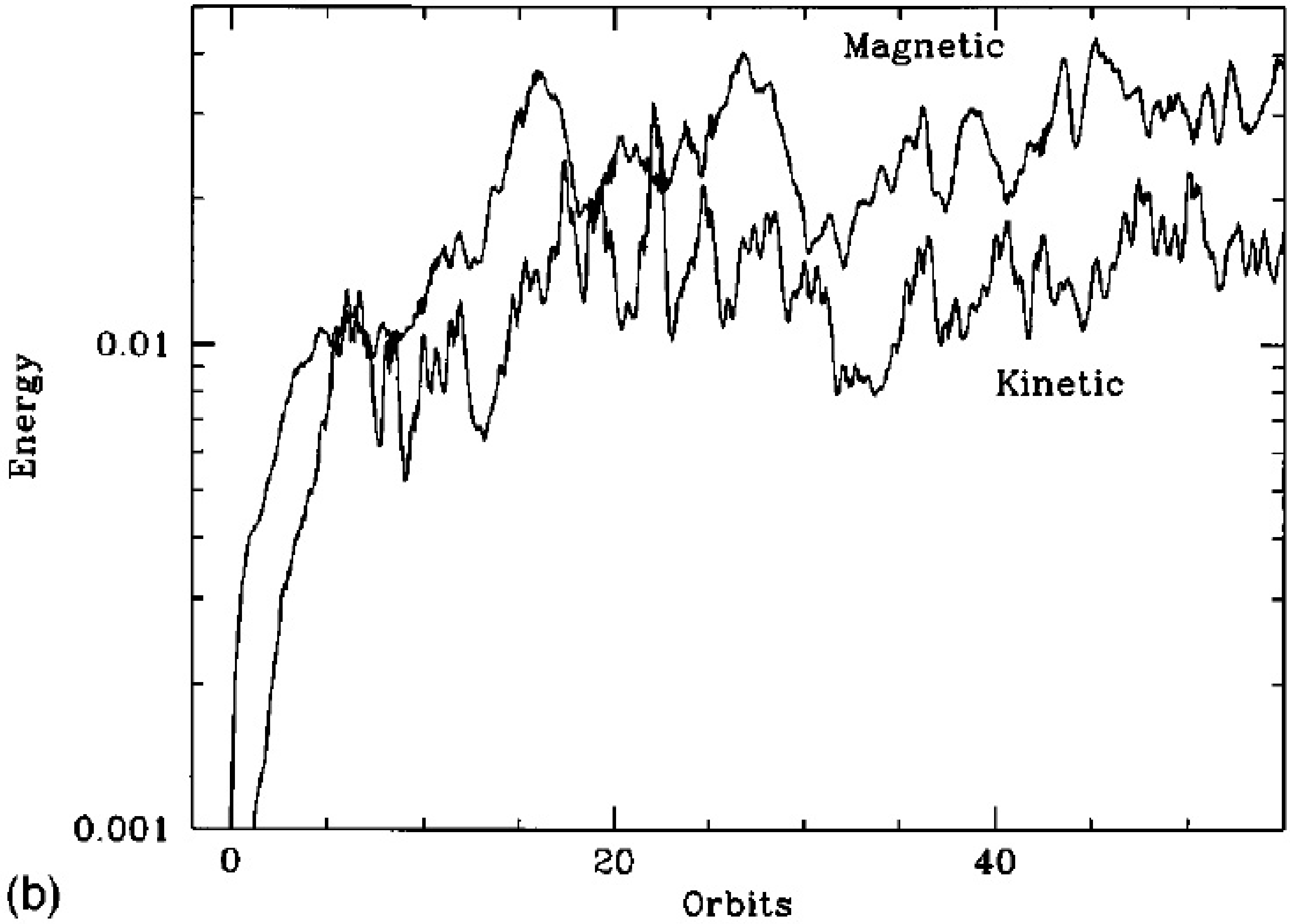
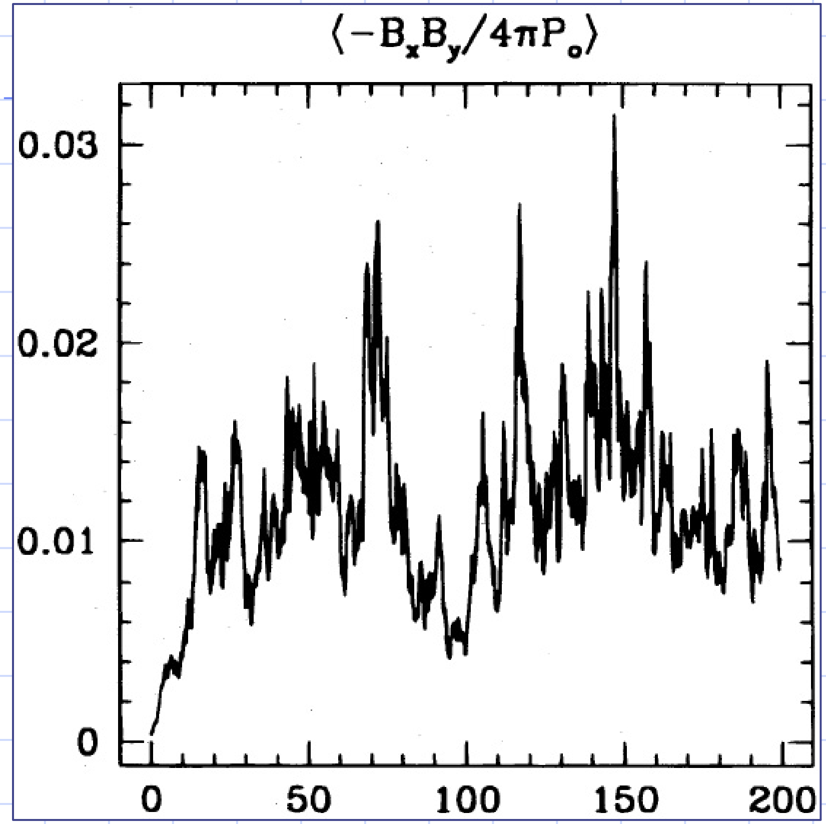
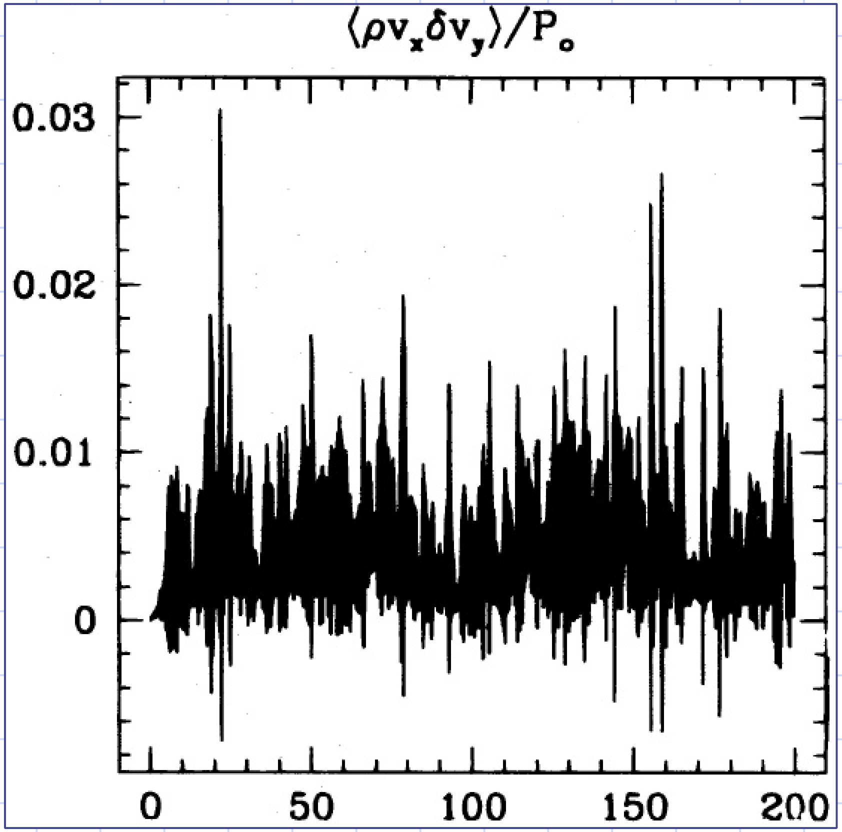
*Astrophys. Jour. **376**, 214 (1991)

Magnetic Field Saturation [5]



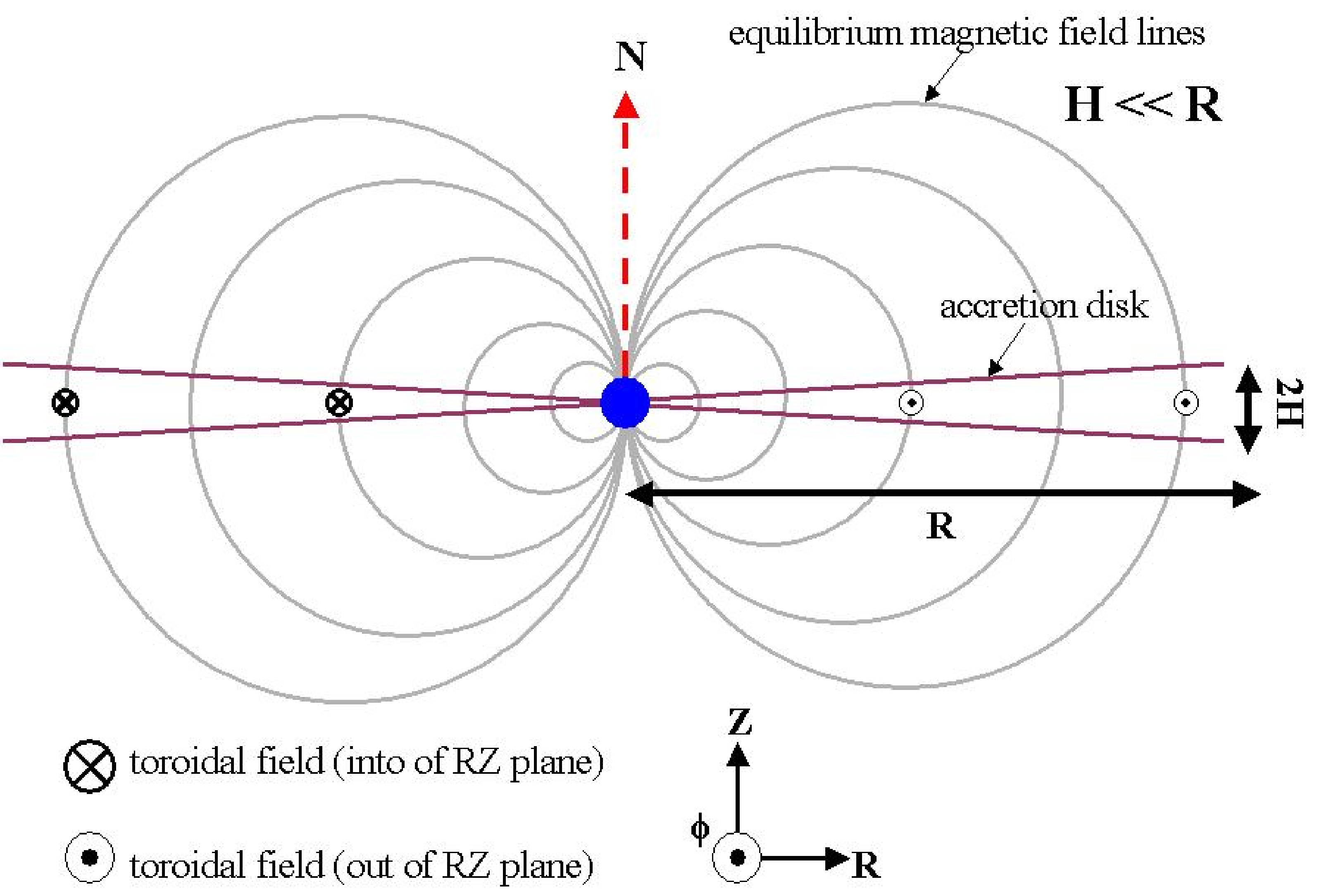
Significant Viscous Stresses [6]



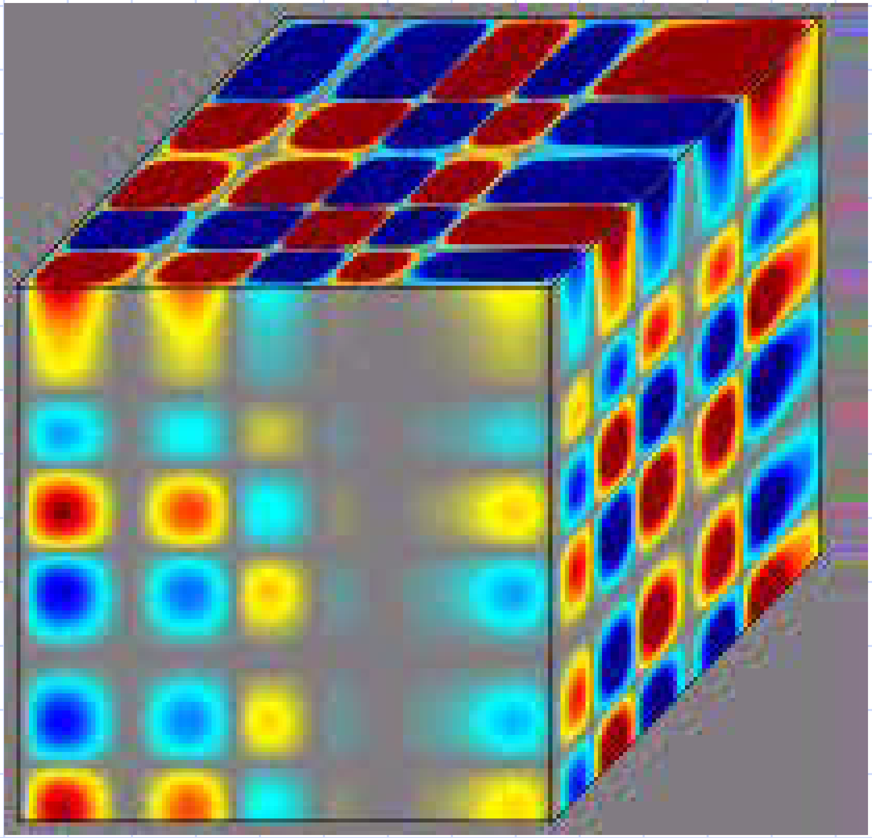


$\alpha \sim 10^{-2}$ in above simulations

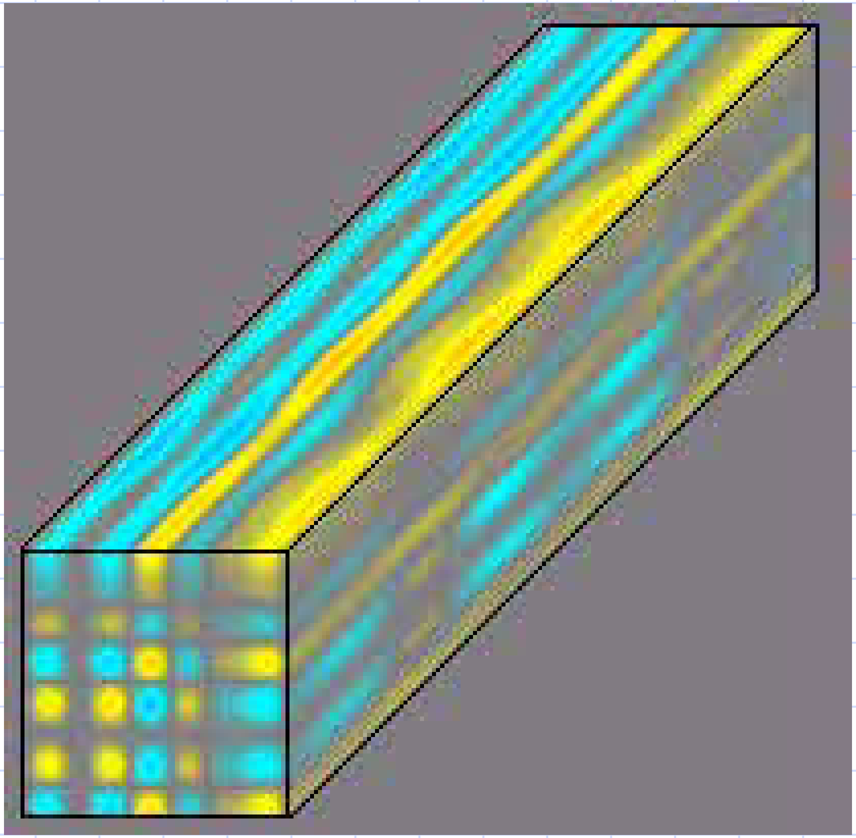
The MRI in accretion disks



Nonlinear Simulations I



no magnetic fields



with magnetic fields

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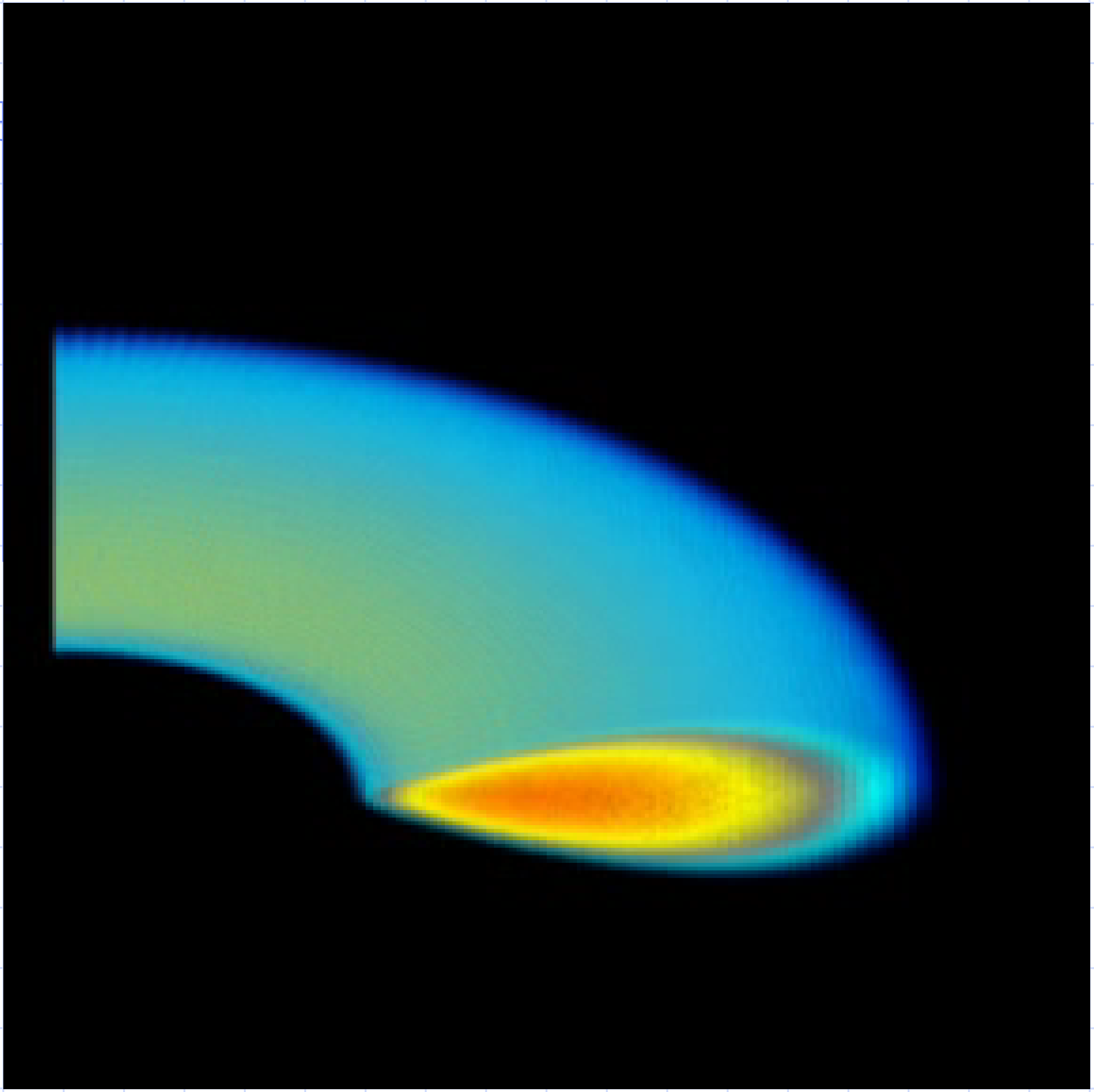

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The MRI is simply one manifestation of how magnetic fields modify disk stability

Instability Conditions

	nonmagnetized	magnetized
angular momentum transfer	angular momentum ($\Omega^2 R$) decreases radially outwards	angular velocity (Ω) decreases radially outward or upward
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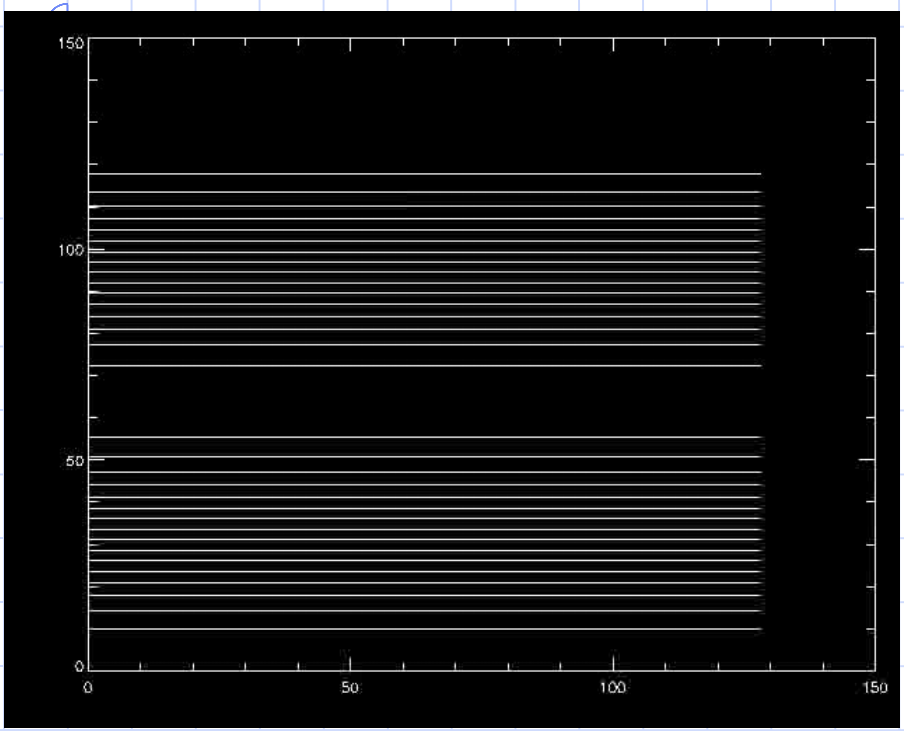
Nonlinear Thermal Instability In Magnetized Plasma



magnetic field lines

temperature

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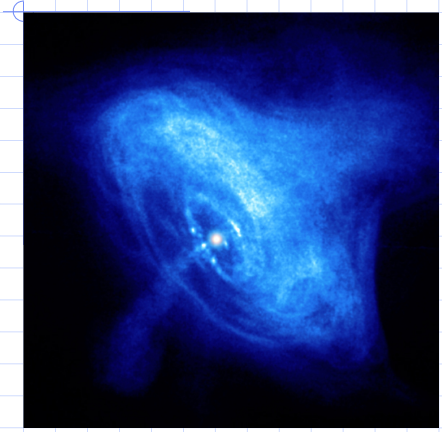

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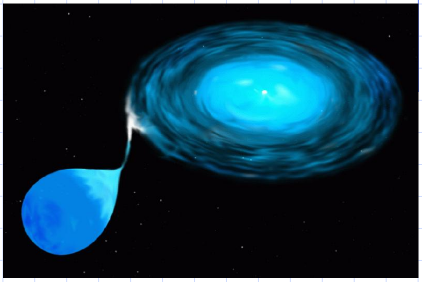

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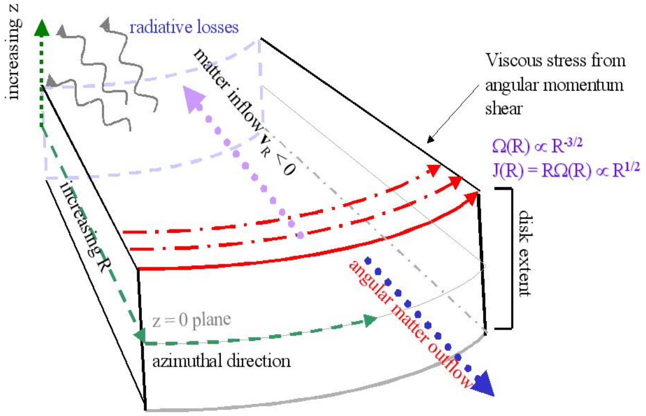

here, the disk wants to rigidly rotate (omega is a constant) – this is what the viscosity does.

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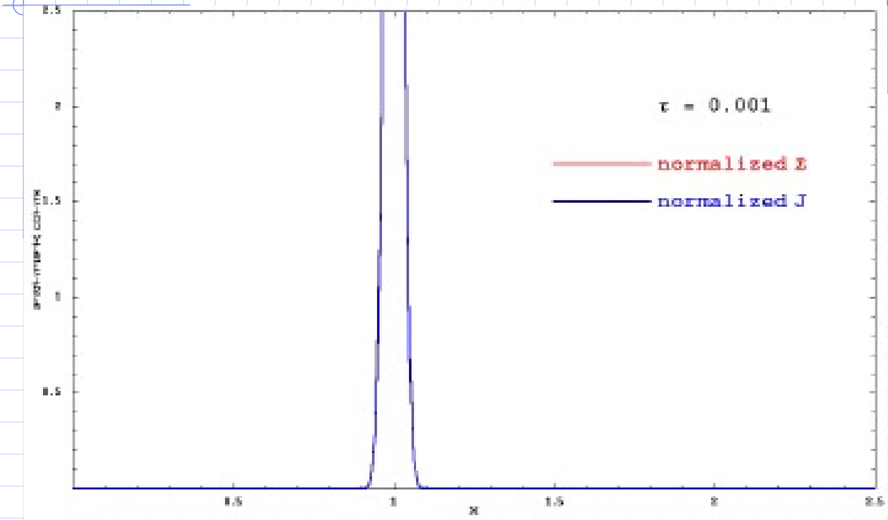

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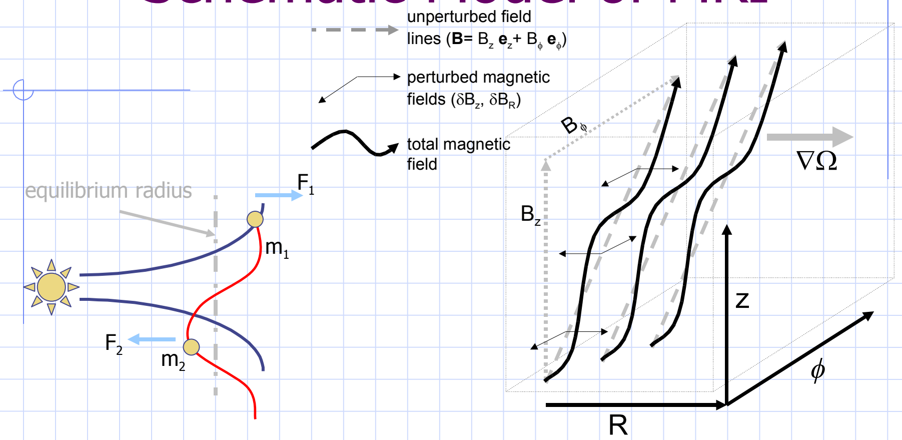

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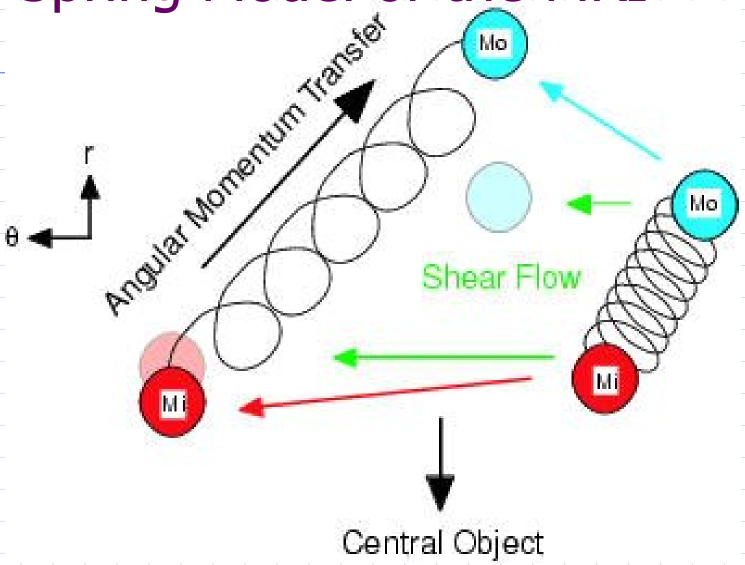

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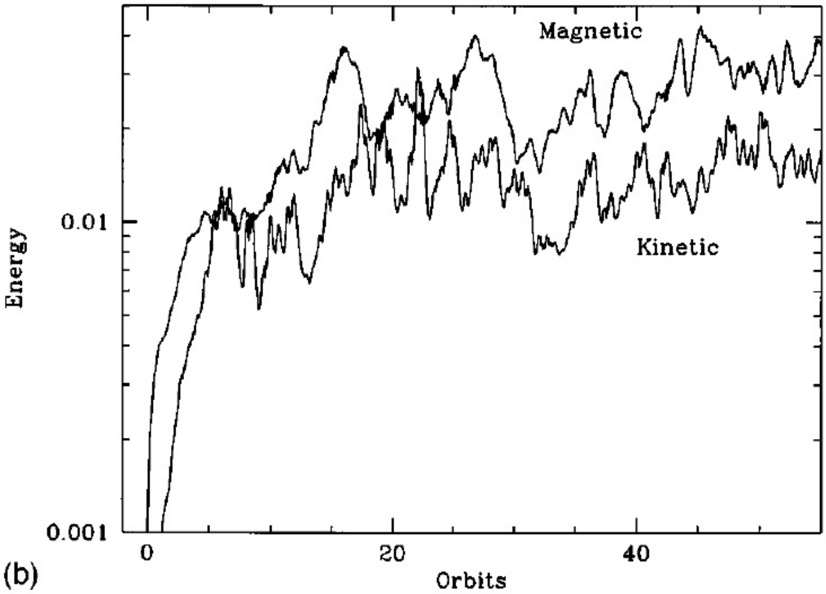

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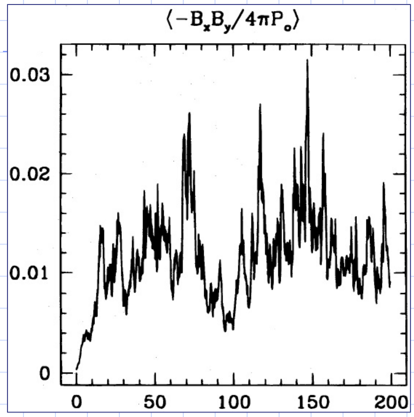
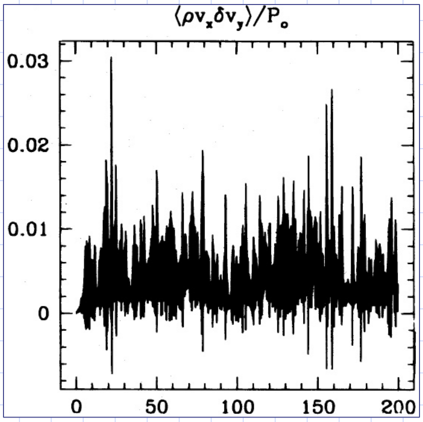


$\langle -B_x B_y / 4\pi P_o \rangle$



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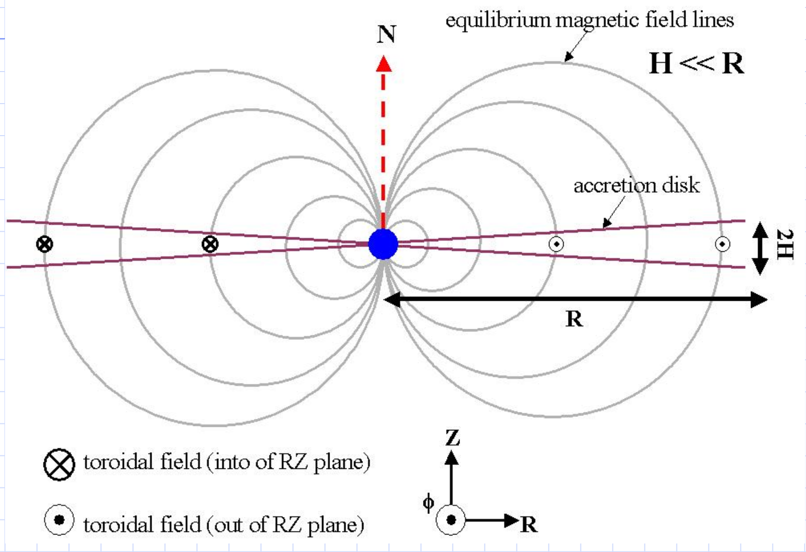

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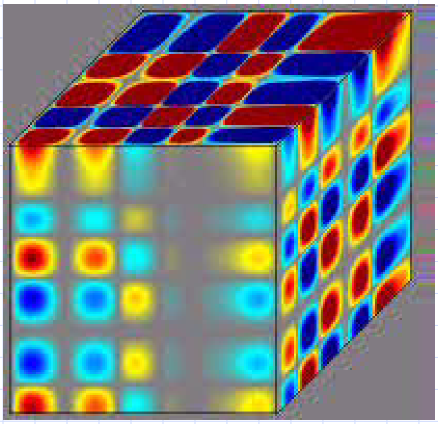

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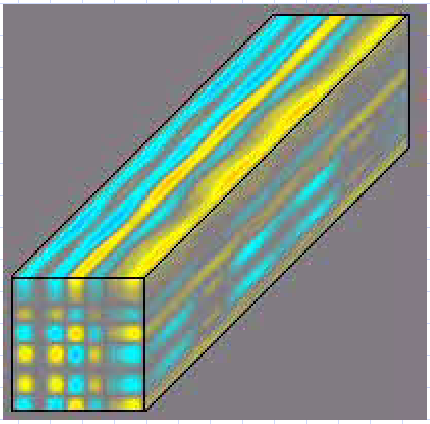

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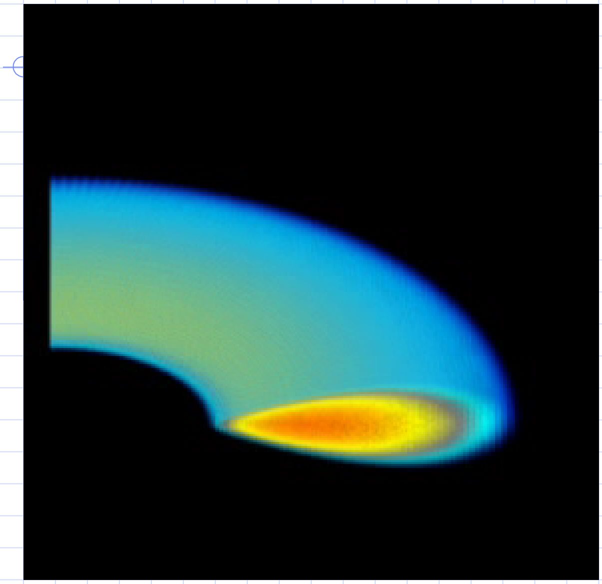

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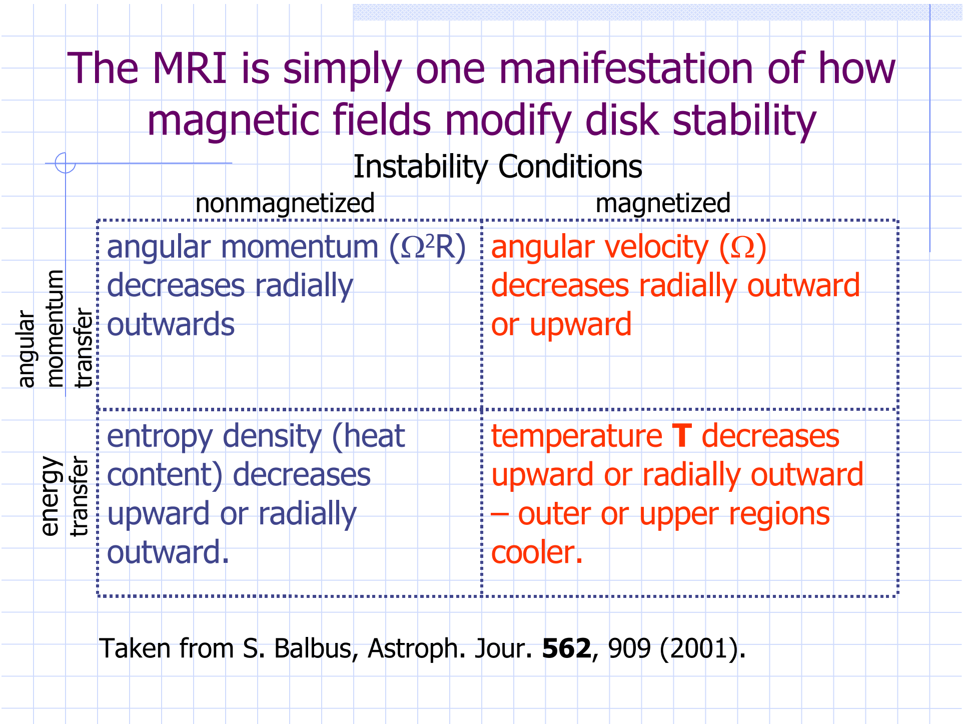

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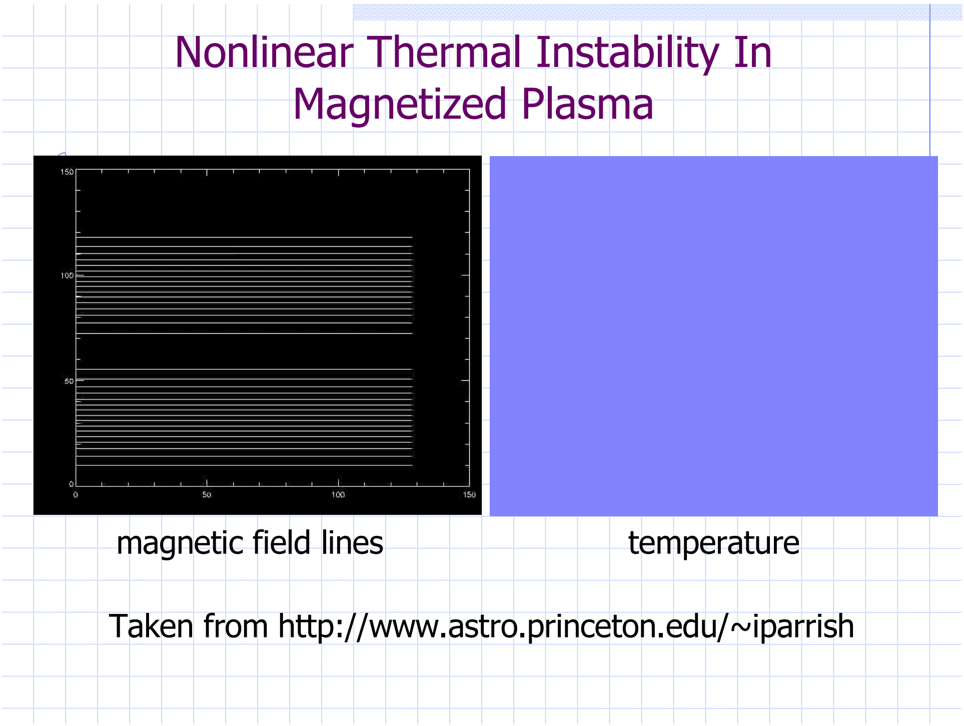
Go into these conditions in more detail, namely how they relate to the direction of the gravitational acceleration. Also these are stability criteria within the disk



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Note that this is a plasma that is stable under the schwarzchild criterion.