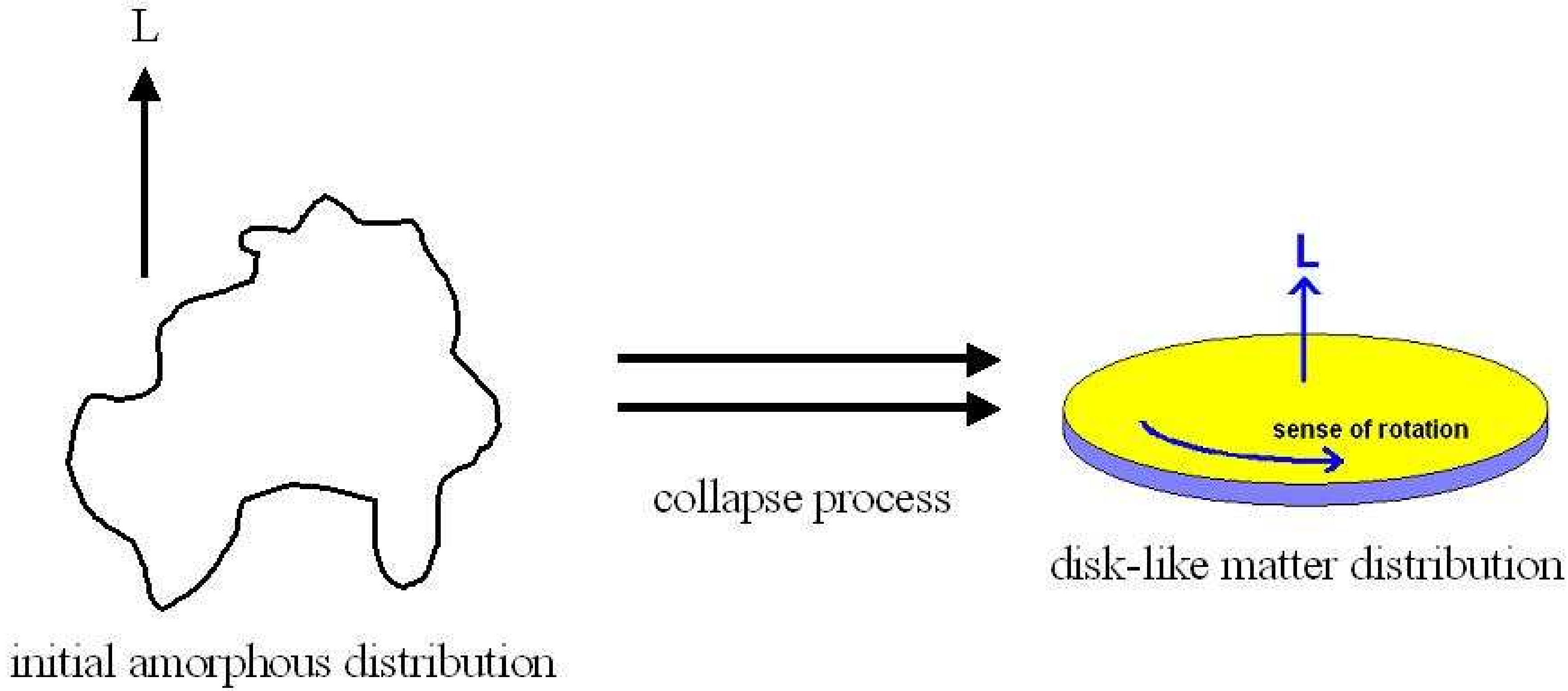
# A Non-Viscous Transport of Angular Momentum in Accretion Disks

T. Islam and B. Coppi

The collective modes which can transport angular momentum in thin accretion disks have been shown[1] to be of the "particle mixing" type[2]. in that they do not involve particle transport. An important difference from the effects of collisional or turbulent viscosity is that these modes do not convert thermal energy into thermal energy. Therefore, commonly made assumptions on the processes that can heat accretion disks are questioned. The quasilinear theory of these modes is reviewed[3] and a relevant model accretion equation is proposed and solved.

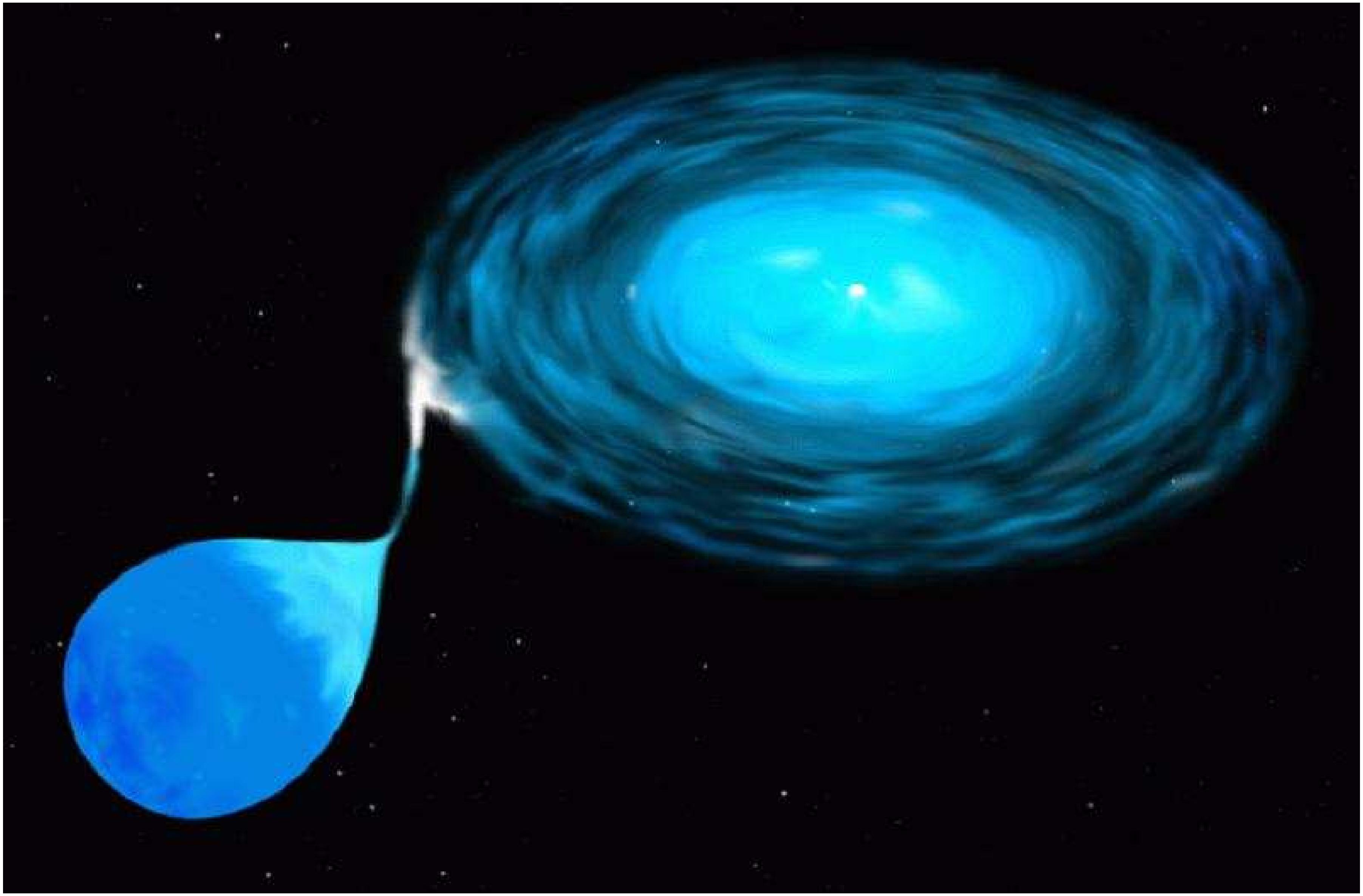
# Background and Description of Accretion Disks

Matter collapsing from an initial "lumpy" distribution will coalesce into a disk, due to the conservation of angular momentum.



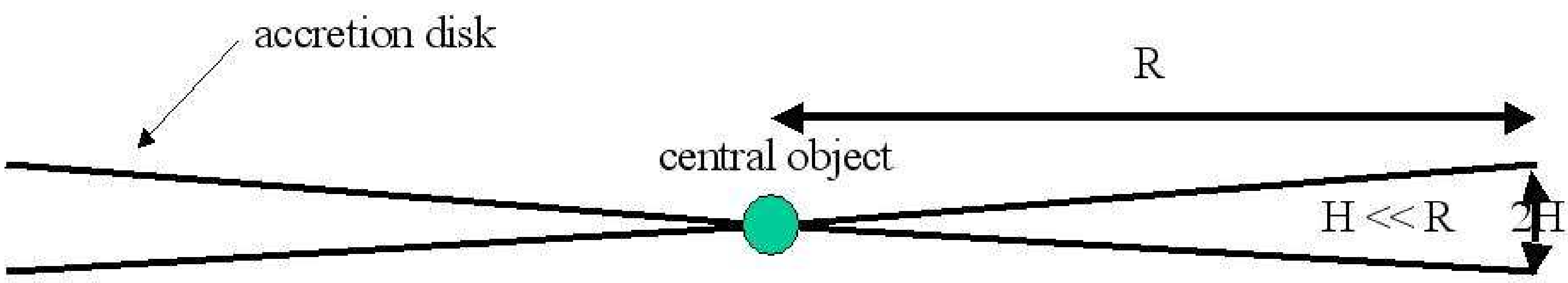

The collapse occurs along a direction parallel to the spin axis (perpendicular to the plane of the disk).

*In general it is easier to lose kinetic energy than it is to lose angular momentum, hence the preponderance of disk-like structures in the universe (i.e. galaxies, protostellar disks, accretion disks).



Artist's conception of an accretion disk in a binary system around a compact central object. A steady outward flow of angular momentum and inward flow of particles may develop.

# Model of the Thin, Ionized Non-Self Gravitating Accretion Disk




- gravity dominated by the central object, So $\Omega \propto R^{-3/2}$, where $\Omega$ is orbital frequency R is disk radius
- Thin accretion disk (H << R), so $v_{th} << v_{\phi}$, where H is disk height $v_{th}$ is thermal velocity $V_{\phi} = R\,\Omega$ is orbital velocity
- Radial magnetic fields $B_R << B_{\phi}, B_z$.
- Largely ionized disk and long mean free paths – justifying collisionless MHD equations throughout the disk.

# Magnetorotational Instability

The magnetorotational instability (MRI)[4, 5] was reintroduced to magnetized accretion disks[6] as a fast, powerful method to transport angular momentum in accretion disks.

- transports particles and angular momentum in MHD plasma.
- maximum growth rate $\gamma \sim \Omega$.
- wavenumber of fastest modes $k \sim v_A/\Omega$.
- diffusion coefficient $D \equiv k^{-2}\gamma \sim \alpha_{SS} c_s H$[7], where $\alpha_{SS} = v_A^2/c_s^2$ is the *Shakura-Sunyaev* $\alpha$ *parameter*.

## Nomenclature

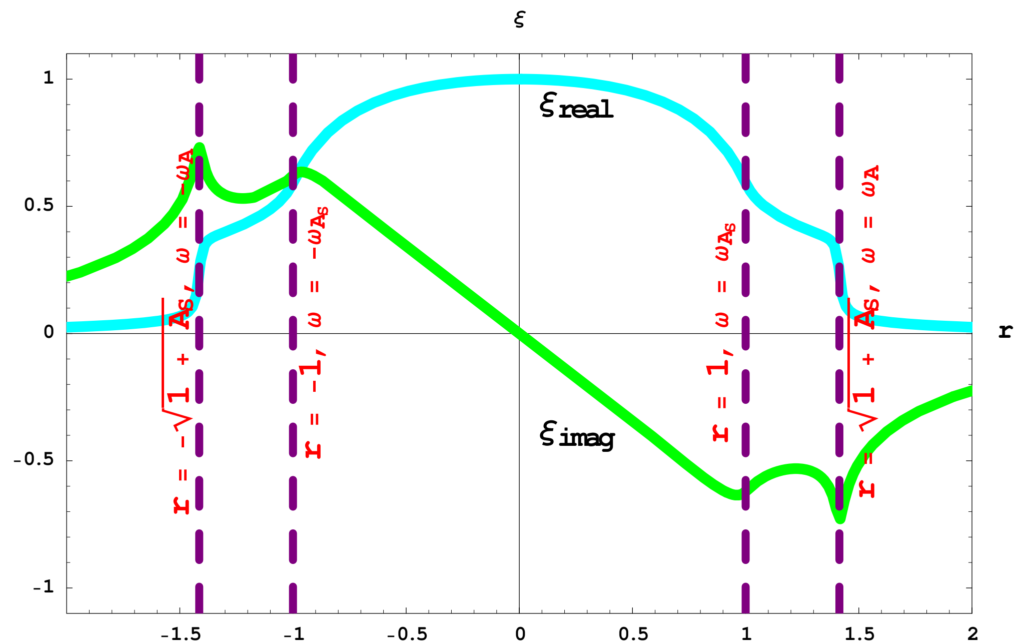
| | | |
|---|---|---|
| $v_A^2 = \frac{B^2}{4\pi\rho}$, Alfvén speed | $c_s^2 \equiv$ sound speed. | $\lambda_0 = \frac{\gamma_0}{\omega_{A_s}}$ |
| $A_s = \frac{v_A^2}{c_s^2}$ | $\omega_A = k_\parallel v_A$, shear Alfvén wave | $\omega_{A_s} = k_\parallel \sqrt{\frac{v_A^2 c_s^2}{v_A^2 + c_s^2}}$, magnetosonic wave |
| $\alpha_k = \frac{2}{3}\left\lvert\frac{d\ln\Omega}{d\ln R}\right\rvert$, rot. shear | $\alpha_z = \frac{3B_\phi}{2B_z}\alpha_k$, | $\delta_0 = \frac{\omega_{A_s}}{\lvert n^0\Omega'\rvert} = -\frac{\omega_{A_s}}{\Omega k_z \alpha_z}$, |
| $r = \frac{R - R_0}{\delta_0}$, | $K_s = \frac{\omega_{A_s}}{b_\phi \Omega}$ | $D_T = D_\mu + \frac{D_m c_s^2}{c_s^2 + v_A^2} + \frac{D_p v_A^2}{c_s^2 + v_A^2}$ total diffusion coeff. |

# Justification and Form for 3D (Nonaxisymmetric) Modes

- For thin disks, the appropriate instabilities are ballooning modes: [3, 8, 1, 9] $\hat{v}_R \simeq \tilde{v}_R(z) \exp\left(\gamma_0 t + i k_R (R - R_0)\right)$.
  - they are characterized by $\Delta_R \ll \Delta_z \ll H$, disk height.
  - This requires that $v_A^2 \ll c_s^2$, limiting applicability.
  - Properly localized mode (i.e. mode decreases as quickly in $z$ or quicker than density) with *discrete* eigenmodes (discrete values of $k_R$, $\gamma_0$); therefore, it is difficult to construct radially localized packets.
- Axisymmetry imposes a strict condition between $k_R$ and variation in $z$ – since the disk is *thin*, the width of modes vertically $\Delta_z < H$, and $k_R$ must be very large.
- Introducing nonaxisymmetric modes – azimuthal dependence – allows for an extra degree of freedom. One can choose that combination of total wavenumber in $\phi$ and $z$ such that magnetic field lines are minimally bent, i.e. $\mathbf{k} \cdot \mathbf{B} \ll kB$. Form of the 3D modes:

$$\hat{A}(R, z, \phi, t) = \tilde{A}(R) \exp\left(i k_z z + i n^0 \phi - i \omega t\right)$$

# Mode Properties[3]

- Eigenmodes with eigenvalues $K_s$ are found for which $\mathrm{Re}\,\tilde{\xi}$ is even, $\mathrm{Im}\,\tilde{\xi}$ is odd.
- These modes are excited for relatively large magnetic energy densities, $v_A^2 \sim c_s^2$.
- Mode is localized over distances $\delta_0 = \omega_{A_s}/\left(\Omega k_z \alpha_z\right) \sim k_\parallel H/k_z \ll R$ about the corotation point, where $n^0\Omega(R) = \mathrm{Re}(\omega)$.
- Small growth rates given by following: $\gamma_0 \sim \omega_{A_s}^{2/3}\left(D_T \delta_0^{-2}\right)^{1/3} = \left(D_T k_z^2 \Omega^2 \alpha_z^2/2\right)^{1/3} \ll \omega_{A_s}$.
- The width of the transition layers: $\Delta_1 = \left(D_T/\left(2k_z\Omega\alpha_z\right)\right)^{1/3} \ll \delta_0$.
- 


In the MHD approximation to the mode, singularities appear at radii $R - R_0 = \pm\delta_0$ (doppler shift $\bar{\omega} = \pm\omega_{A_s}$) and at $R - R_0 = \pm\delta_0\sqrt{1 + A_s}$ (doppler shift $\omega = \pm\omega_A$).