Accretion in Dilute Magnetized Plasmas: Main Issues, Physics, and Analytic Approaches

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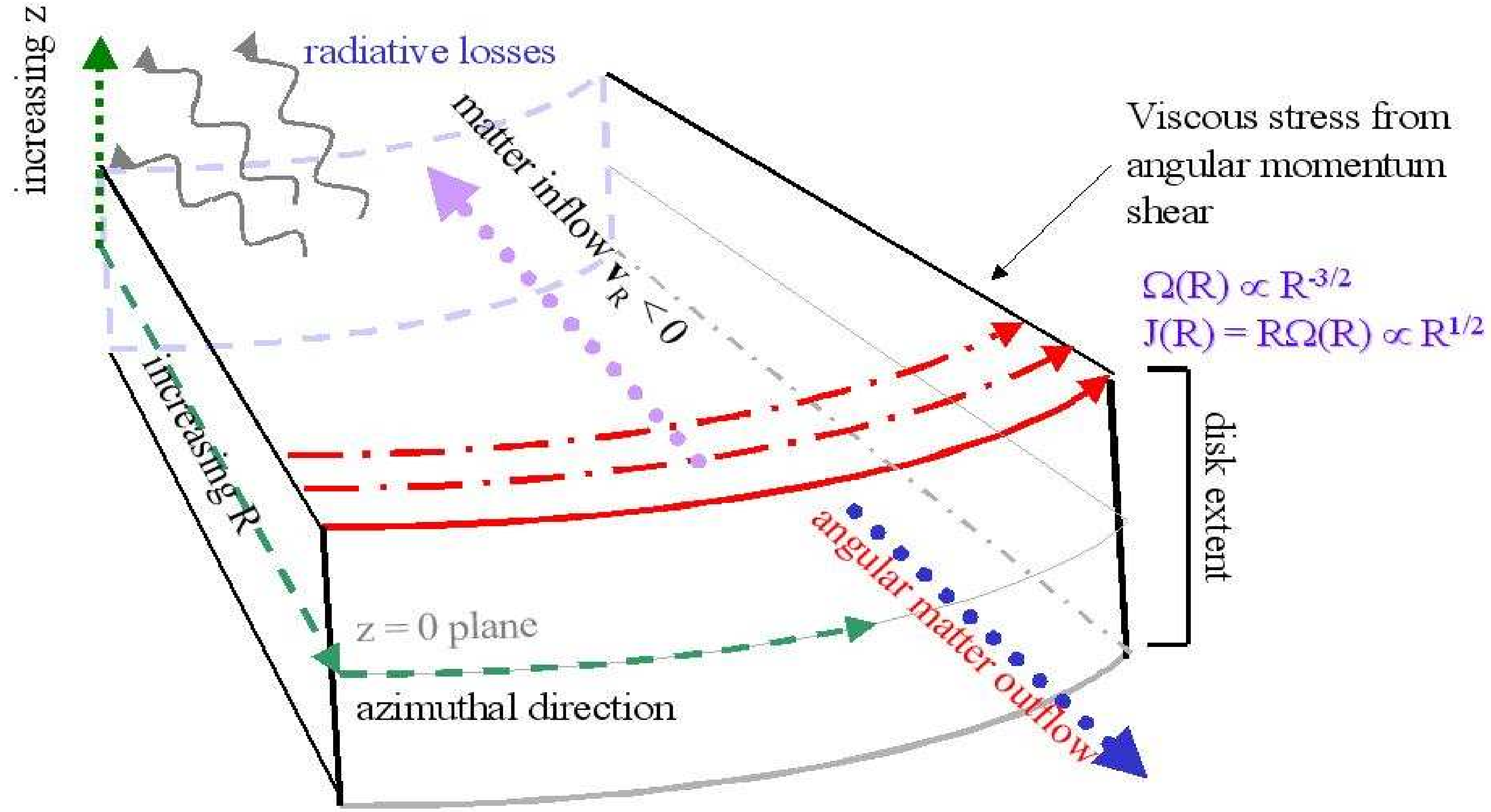
Overview

- Overview of classical accretion disk theory with modifications for dilute plasmas.
- Evidence for very dilute accretion astrophysically.
- Summary of the magnetoviscous and magnetothermal instabilities.
- A more appropriate method of attack for collisionless plasmas: using a fluid-kinetic approach employing the drift-kinetic equation described in Kulsrud (1983); Kulsrud (2005).

Accretion Flow as an Engine



Features of Standard Accretion Disk

In the literature (see, e.g., Pringle (1981); Frank et al. (2002)), this refers to

- **dense**: extremely high Reynolds number disks, collisional viscosity and thermal conductivity cannot determine the accretion dynamics.
- **optically thick**: Energy generated through viscous stresses is radiated away, resulting in:
  - Extremely efficient – classical disks radiate half the available gravitational energy.
  - can be relatively cool on their edges (thermal speeds $\ll$ orbital speeds), making many of them geometrically thin.

Astrophysical Evidence for Dilute Flows: Sag. A

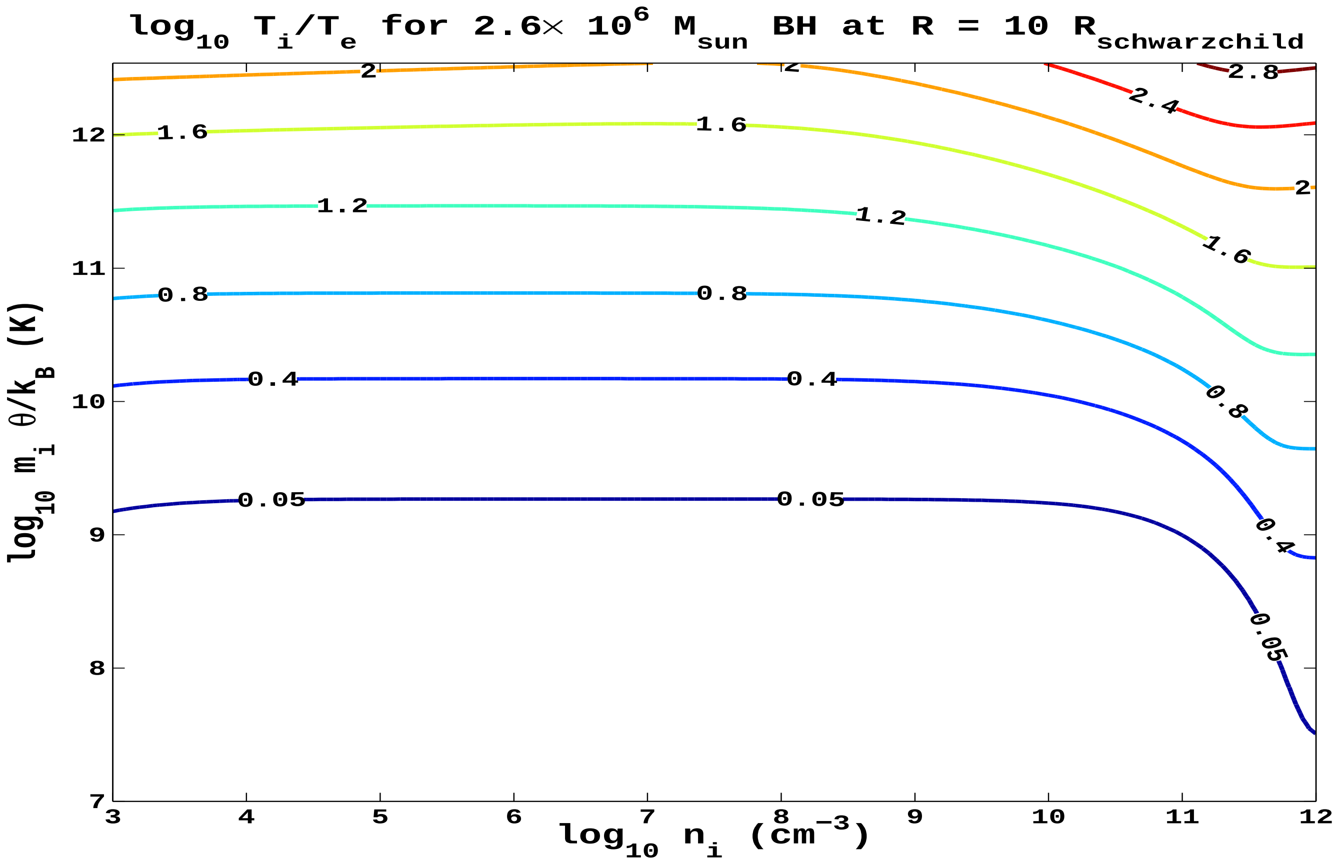
Unlikely to be thin-disk accretion, described in Pringle (1981); Frank et al. (2002):

- From Chandra Baganoff et al. (2003) of $n \approx 10^2$ cm^{-3}, $T \simeq 2$ keV within 0.04 pc (1"), $2.6 \times 10^8 M_\odot$ black hole Schödel et al. (2002); Ghez et al. (2003) implies Bondi accretion rates $\sim 10^{-5} M_\odot$ yr^{-1} Quataert (2003).
- Luminosity of Sag. A $L \sim 10^{36}$ erg s^{-1} $\ll\sim GM\dot{M}/R_{\text{inner}} \simeq 10^{41}$ erg s^{-1}.
- Mass of central black hole implies peak emission, from thin disk accretion models, at the infrared. Spectral measurements about the peak imply accretion rates $< 10^{-10} M_\odot$ Narayan (2002).

Radiatively Inefficient Accretion Flows

- At extremely high temperatures and even for mass accretion close to the Eddington rate, radiation losses become “relatively” inefficient (cooling timescales > accretion timescales) and the plasma becomes **two-temperature**, with a (much) hotter ion component.
- Ions remain at the virial temperature, $\simeq 10^{12}$ K about the central black hole.
- Electrons are heated by electron-ion collisions, and cool through following processes:
  - synchrotron.
  - bremsstrahlung.
  - Synchrotron and bremsstrahlung self-compton.

Two-Temperature Nature in Dilute Plasmas



$\theta = k_B \left(T_i + T_e\right)/m_i$; T_i and T_e are ion and electron temperatures.

Nature of Astrophysical Dilute Plasmas

- Plasma mean free path between collisions is on the order of the system scale size or shorter.
  - fluid approximation breaks down if collisionality is too mild – no local thermodynamic quantities such as temperature b/c no thermodynamic equilibrium.
  - where fluid approximation is valid, a fluid analysis demonstrates that viscosity and thermal conductivity become important in characterizing plasma stability.
- Plasmas remain strongly magnetized, in which ion cyclotron frequency $\gg$ collision frequency, or any other inverse time scales of plasma dynamics.
  - viscosity and thermal conductivity become anisotropic – directed along magnetic field lines.

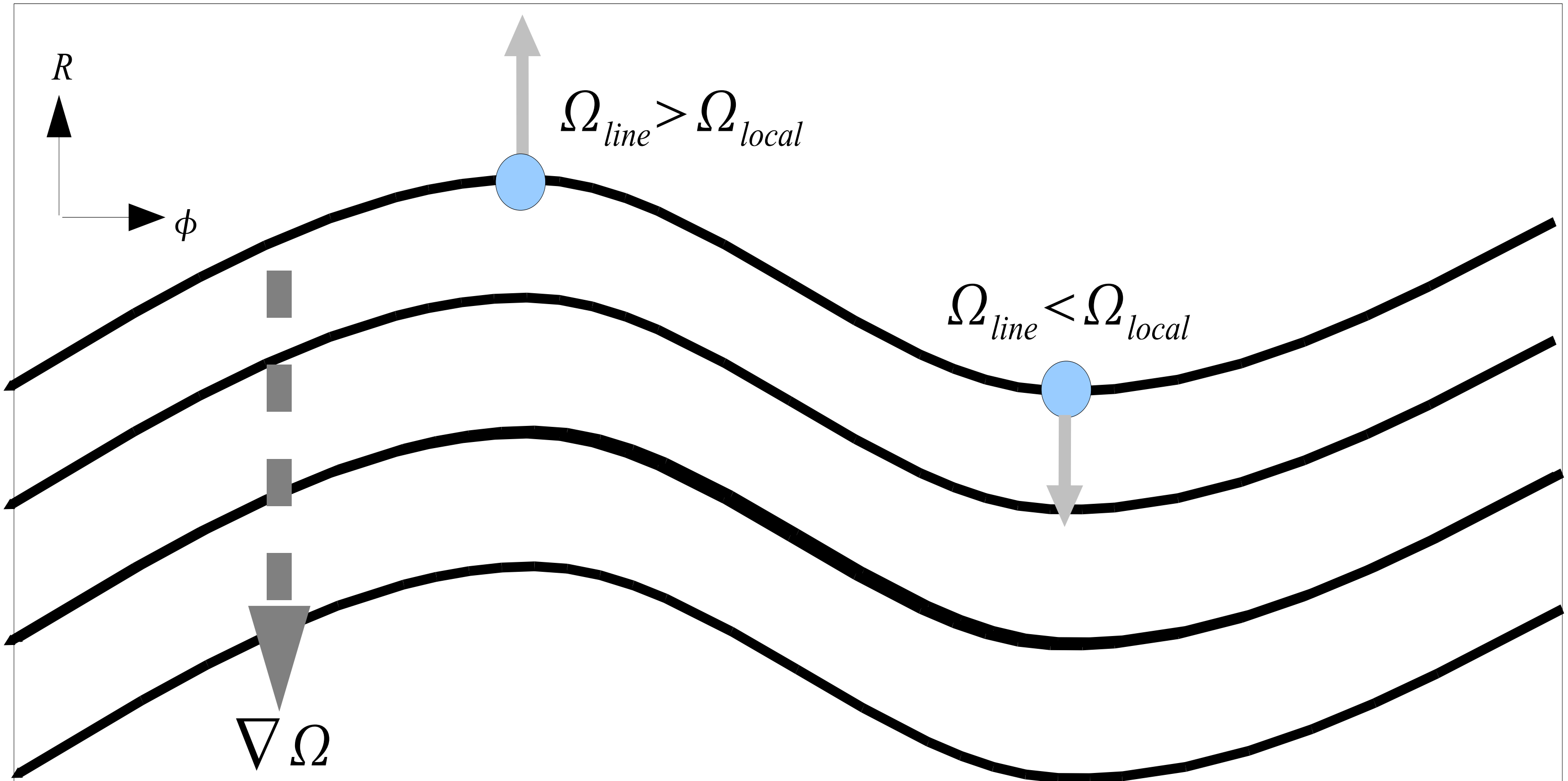
Main Modifications to Accretion Disk Dynamics

- **radiatively inefficient**: one must take into account at least turbulent heat fluxes (see, e.g., Balbus (2004)), and perhaps large transport terms, to transport or dissipate viscous energy.
- **dilute/collisionless**: the MRI can destabilize a standard accretion disk; in a dilute plasma, it is thermal conductivity (magnetothermal instability) and viscosity (magnetoviscous instability) along magnetic field lines that provide the channels by which the plasma is destabilized.

Energy Balance in Accretion Without Diffusive Terms

$$
\nabla \cdot \left(\frac{\gamma k_B \rho}{m_i (\gamma - 1)} \langle \delta v_R (\delta T_i + \delta T_e) \rangle \mathbf{e}_R \right) +
\frac{k_B T_e}{m_i (\gamma - 1)} \langle \delta\rho \delta v_R \rangle \frac{\partial}{\partial R} \ln P_e \rho^{-\gamma} +
\frac{k_B T_i}{m_i (\gamma - 1)} \langle \delta\rho \delta v_R \rangle \frac{\partial}{\partial R} \ln P_i \rho^{-\gamma} = -T_{R\phi} \frac{d\Omega}{d \ln R}
$$

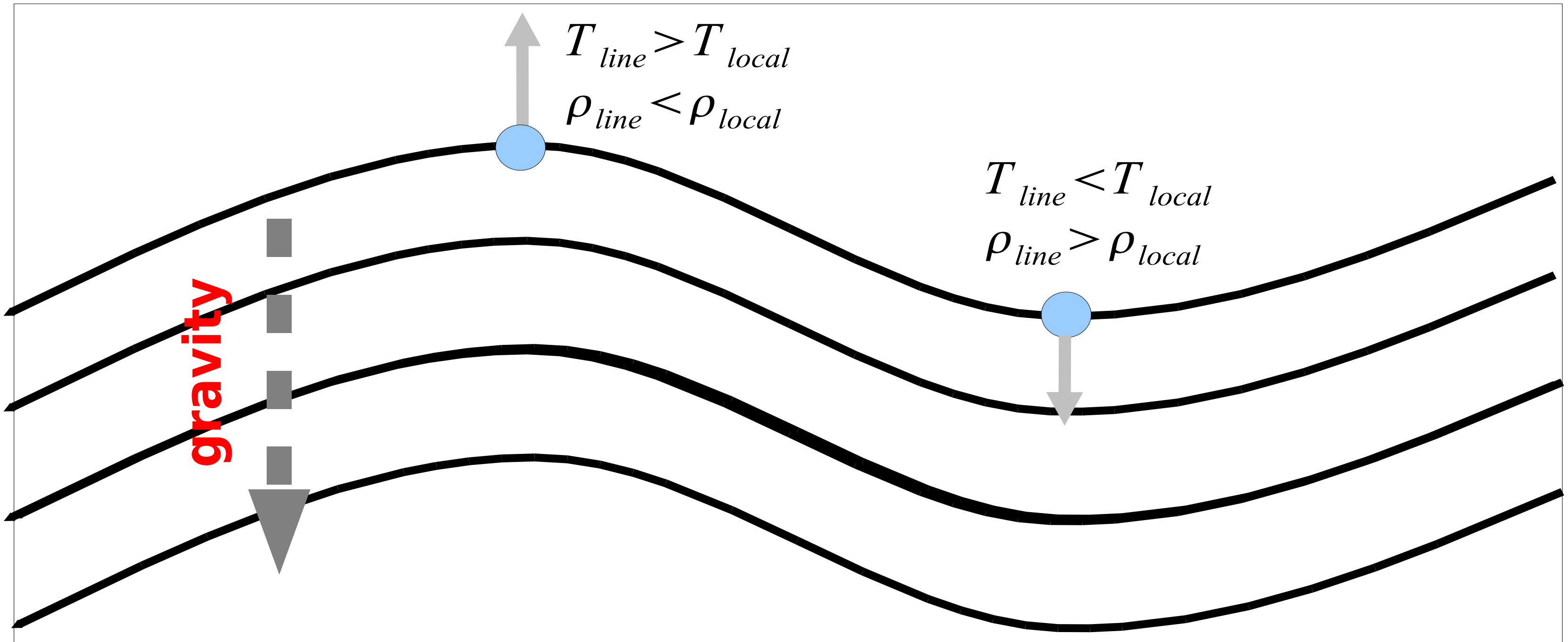
Schematic of Magnetoviscous Instability (MVI)



viscosity ensures constant angular velocity along magnetic field lines

Schematic of Magnetothermal Instability (MTI)

COLD



thermal conduction ensures isothermality along magnetic field lines

HOT

Regimes of Applicability of Fluid Analysis of Dilute Plasma

- The magnetoviscous forces dominate the plasma dynamics under the following condition:

  $$\nu_i/\Omega \lesssim \beta$$

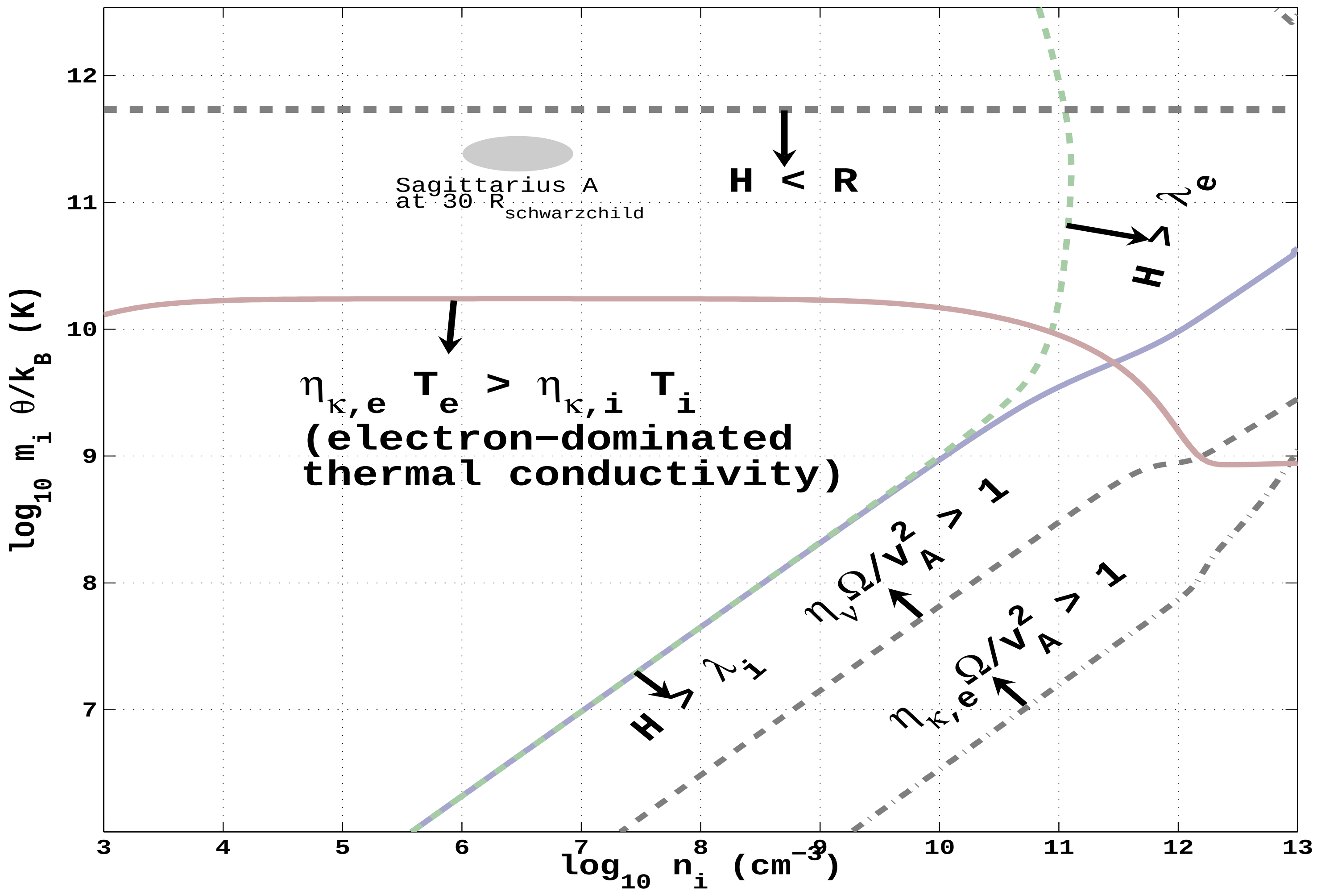
  Where $\beta = 8\pi p/B^2$, ν_i is ion-ion collision frequency, and Ω is orbital angular velocity.

- In the fluid approximation, the electron thermal conductivity dominates. The energy balance equation is affected under the following condition (conditions of magnetothermal instability):

  $$\nu_i/\Omega \lesssim \beta\sqrt{m_i/m_e}$$

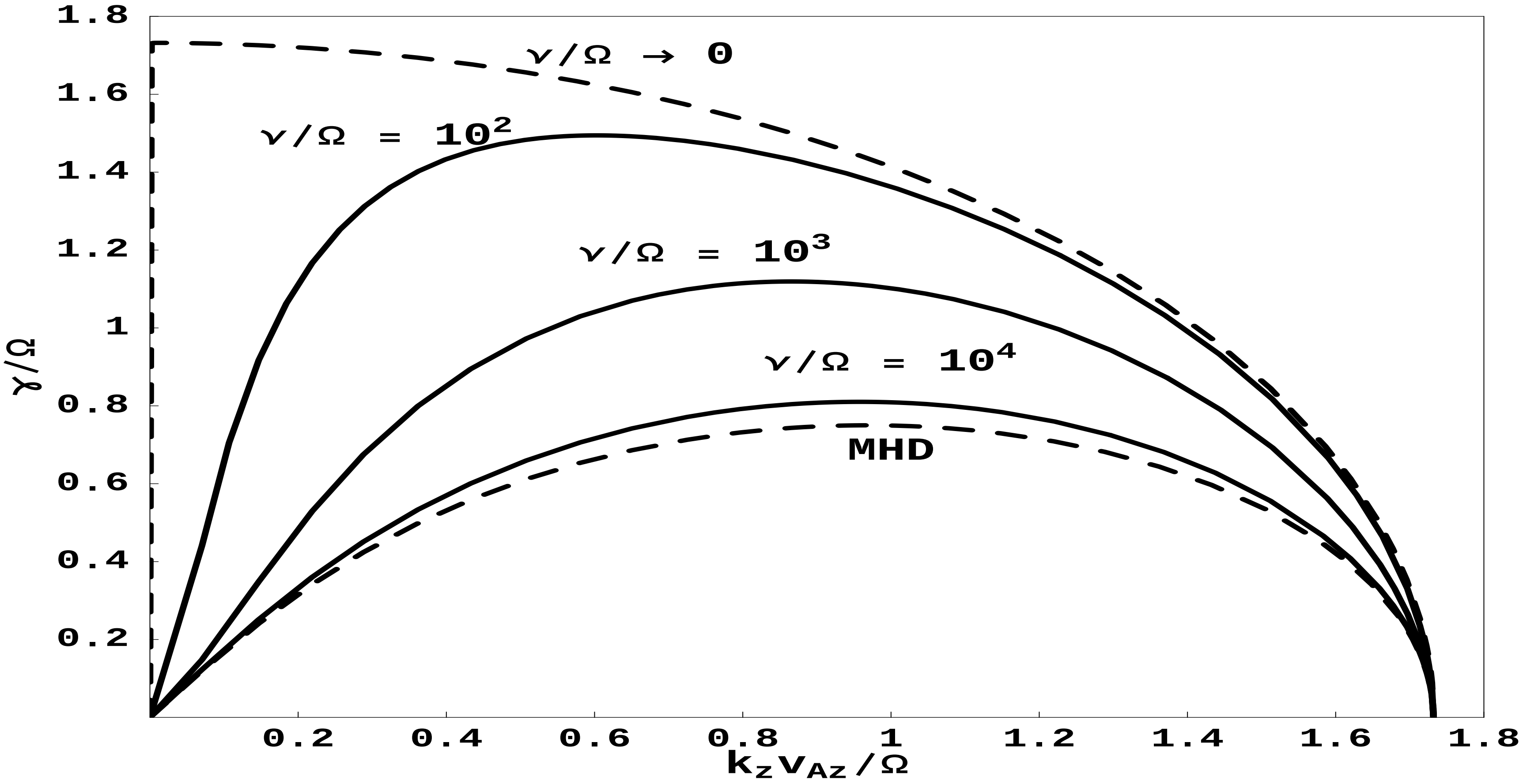
  Where we have used the result, $\nu_e \simeq \nu_i\sqrt{m_e/m_i}$.

- Fluid analysis is bounded by the conditions of short mean free path.


12
11
10
9
8
7
$\log_{10} m_i\ \theta/k_B$ (K)
3
4
5
6
7
8
9
10
11
12
13
$\log_{10} n_i$ (cm^{-3})
Sagittarius A
at 30 $R_{\mathrm{schwarzchild}}$
H < R
$H > \lambda_e$
$\eta_{\kappa,e}\ T_e > \eta_{\kappa,i}\ T_i$
(electron−dominated
thermal conductivity)
$\eta_\nu \Omega / v_A^2 > 1$
$H > \lambda_i$
$\eta_{\kappa,e} \Omega / v_A^2 > 1$

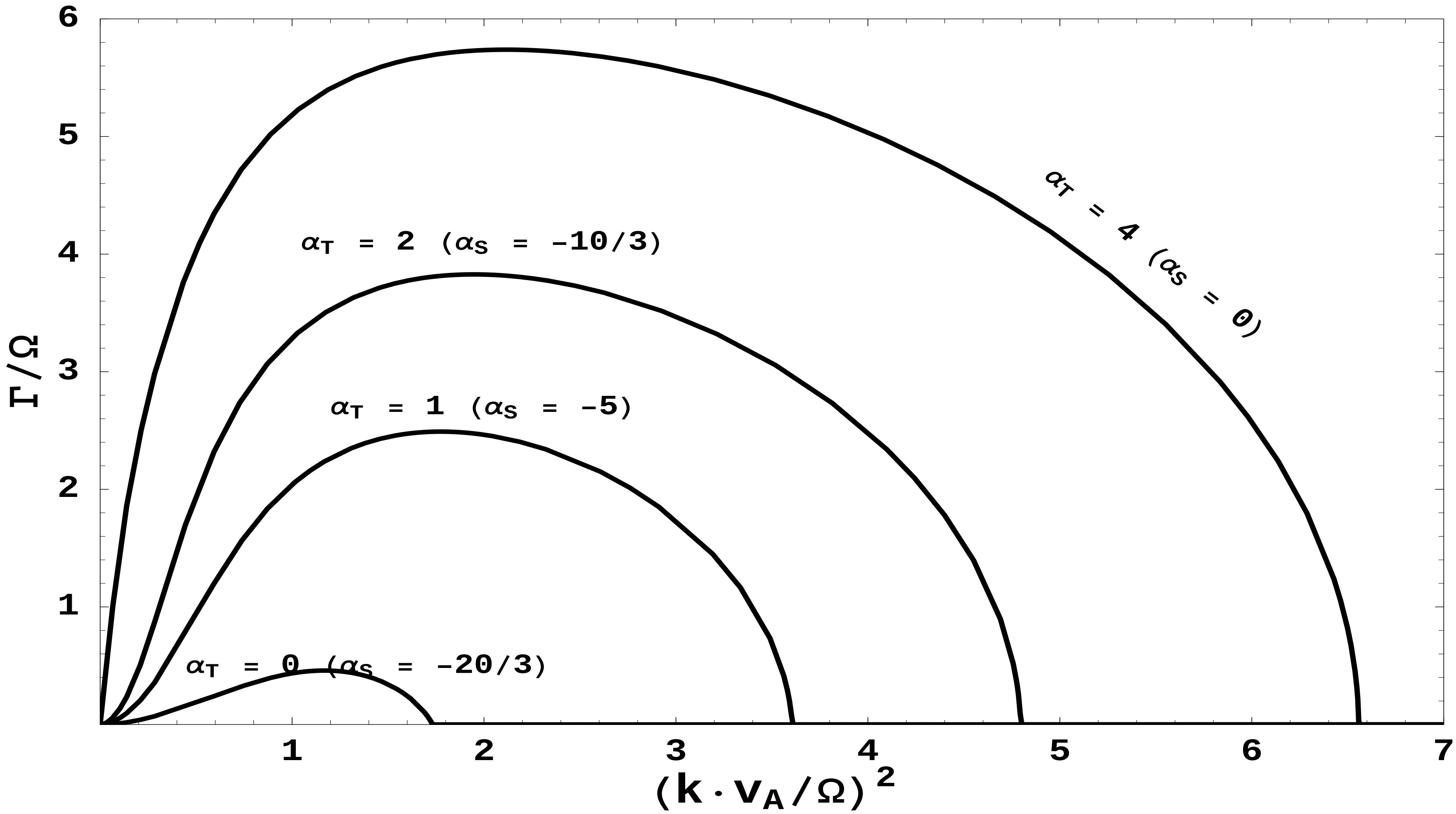
MVI Saturates at wavenumbers

$$k^2 \simeq (\nu_i \Omega) / c_s^2 \lesssim \Omega^2 / v_A^2$$



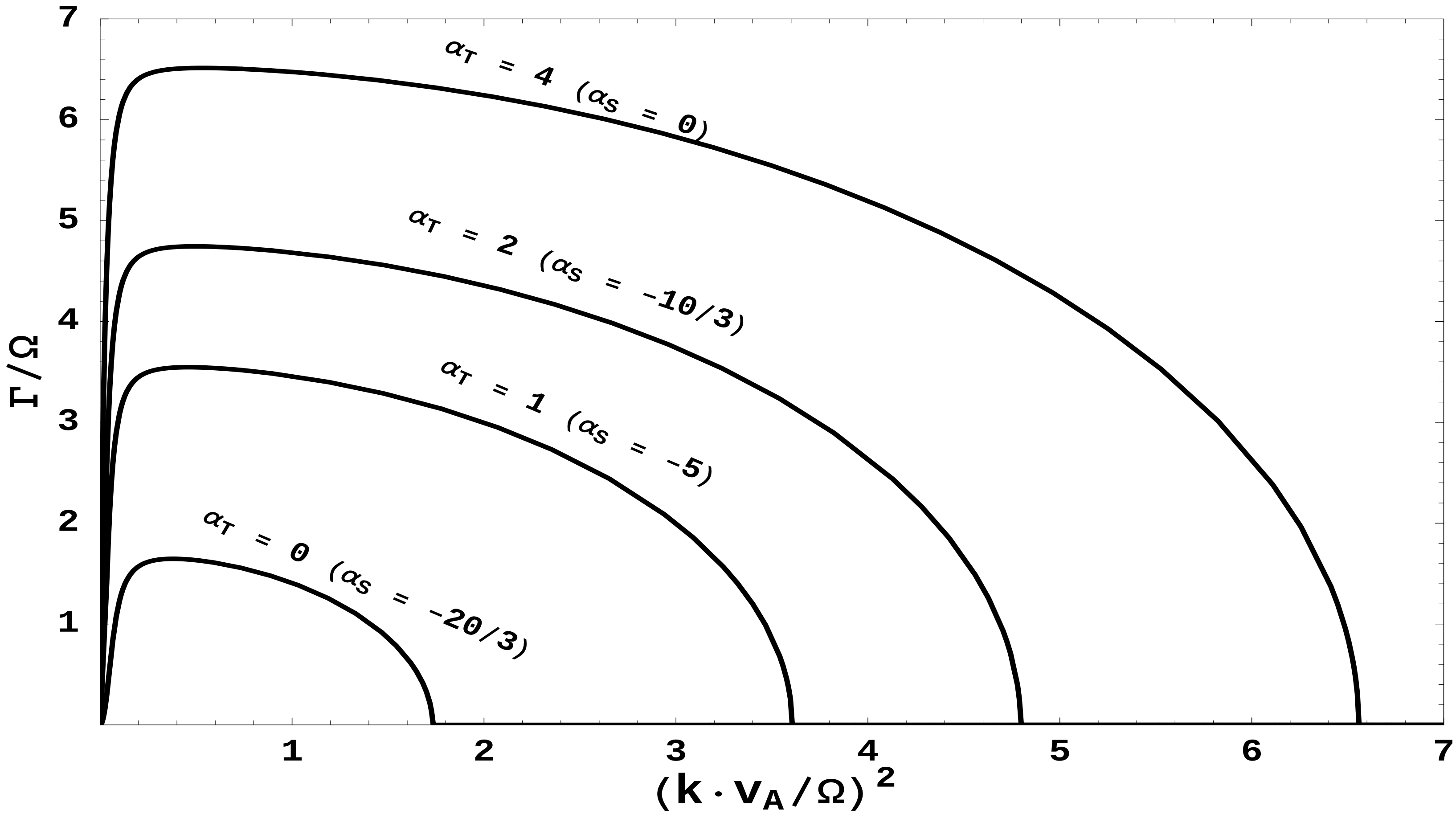
Dispersion relation for MVI, for $\beta = 5 \times 10^3$ plasma, c_s is sound speed.

MRI-analogue of the MTI



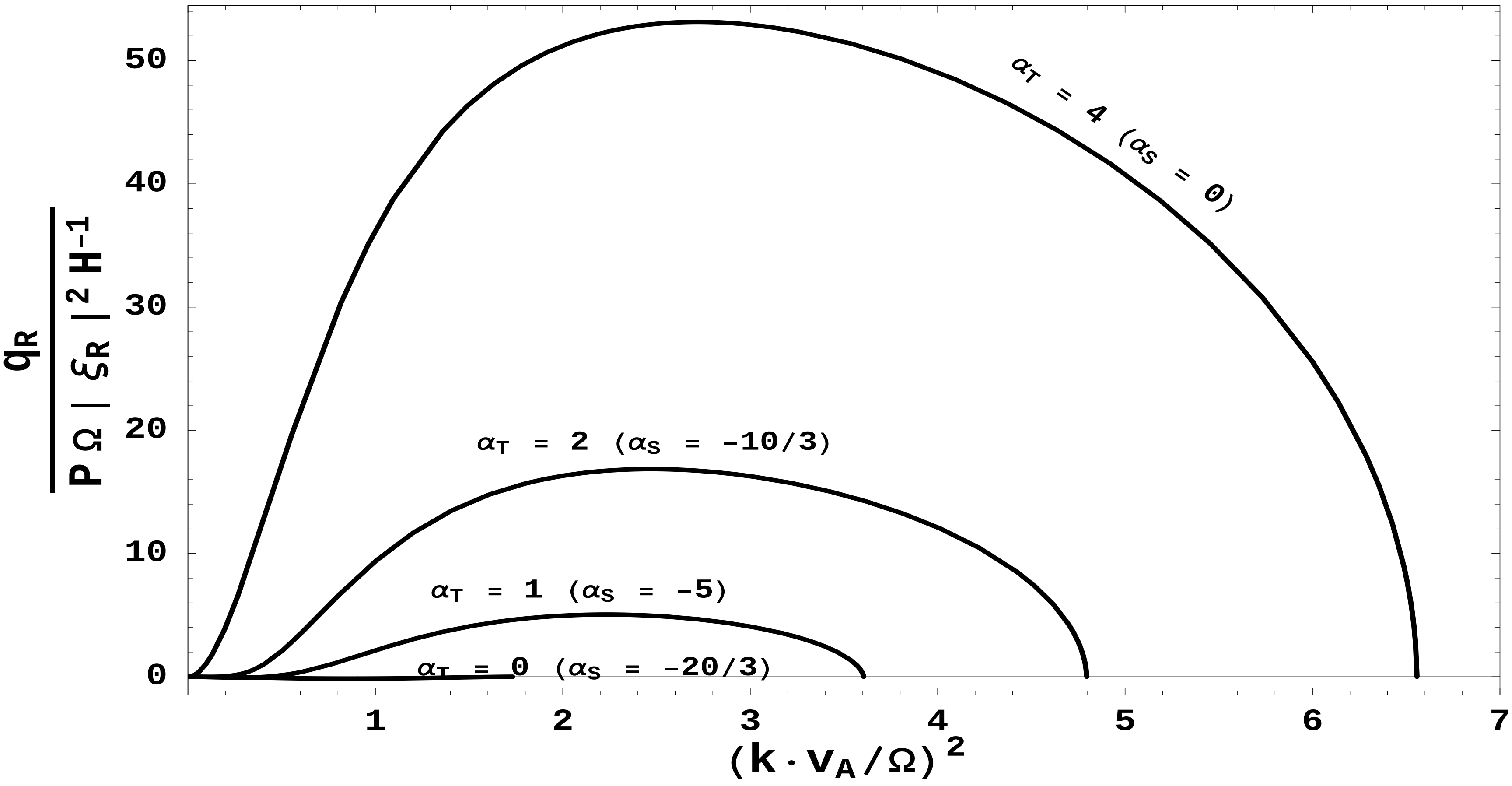
Dispersion relation for small magnetized viscosity, outwardly decreasing pressure and temperatures, convectively stable system.

MVI-analogue of the MTI

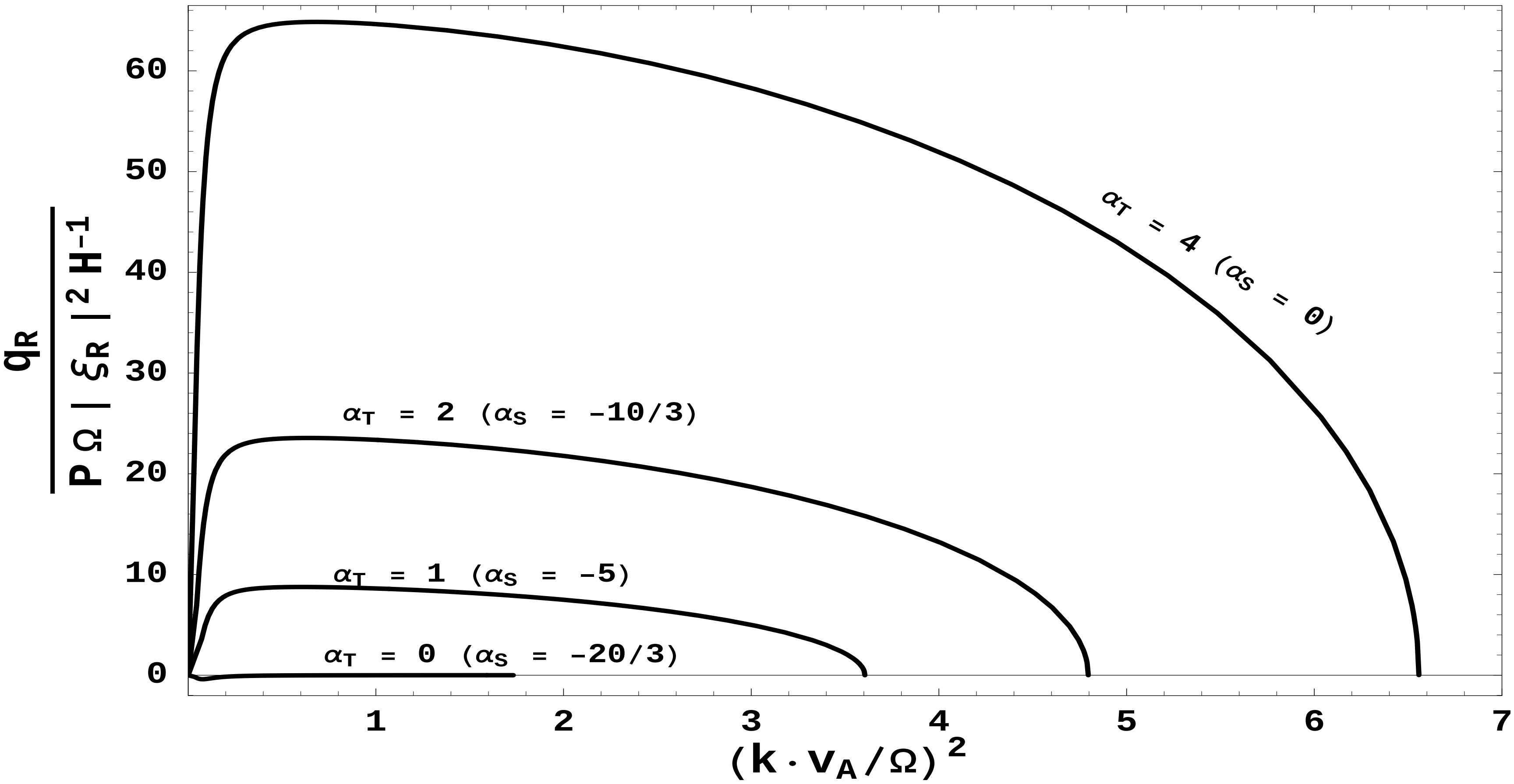


Dispersion relation for large magnetized viscosity ($\Omega/\nu_i \approx 10^3\beta^{-1}$), outwardly decreasing pressure and temperatures, convectively stable system.

Heat fluxes for the MRI-analogue and MVI-analogue of the MTI



normalized heat flux $\rho k_B/m_i \left<\delta v\left(\delta T_i + \delta T_e\right)\right>$ for MRI-analogue unstable MTI modes.



normalized heat flux $\rho k_B / m_i \left\langle \delta v \left(\delta T_i + \delta T_e \right) \right\rangle$ for MVI-analogue unstable MTI modes.

Limits of Fluid Approach

- In the limit of long mean free paths, fluid approach to the problem breaks down – as collisionality is removed, viscosity and thermal conductivity become infinite.
- Appropriate approach is one given by Kulsrud (1983); Kulsrud (2005) – an ordered expansion of the Boltzmann equation in terms of m/e whose velocity moments preserve MHD:
    - non-electromagnetic forces, pressure and heat flux tensors, and mass continuity due to zeroth-order distribution function.
    - quasineutrality: at zeroth-order net electric charge is zero; same bulk velocity: all species have same average velocity.
    - currents and charge distributions arise from first-order distribution functions.

Drift Kinetic Equation

- To lowest order, the distribution function is axisymmetric about the magnetic field, with a drift velocity given by $\mathbf{E} = -\frac{1}{c}\mathbf{v}_E \times \mathbf{B}$.
- To next order, employ axisymmetry using the following change of variables, $\mathbf{b}$ is unit vector along magnetic field.

$$v_\| = \mathbf{v} \cdot \mathbf{b} \text{ (velocity parallel to mag. field)}$$

$$\mu = \frac{(\mathbf{v} - \mathbf{v}_E)^2 - (\mathbf{v} \cdot \mathbf{b})^2}{2B} \text{ (magnetic moment)}$$

$$\tan\psi = \frac{\mathbf{v} \cdot (\mathbf{b} \times \mathbf{v}_E)}{(\mathbf{v} - \mathbf{v}_E) \cdot \mathbf{v}_E} \text{ (phase angle)}$$

And average over ψ.

Final Transform Appropriate to Accretion Problems

- Transform to local rotating frame, ($v_{\|} = \sigma_{\|} + R\Omega b_\phi$, $\mathbf{v}_E = \mathbf{u}_E + R\Omega\mathbf{e}_\phi - R\Omega b_\phi\mathbf{b}$) to get a drift-kinetic equation with extra noninertial (Coriolis and tidal) forces.
- Moment equations, as well as energy balance in accretion disks, reduce to fluid counterparts with addition of collisional operator and sufficiently strong collisionality (effectively, mean free path $<$ system size).
- Provides a framework for more careful stability analysis than has been done in the literature (see, e.g., Quataert et al. (2002); Sharma et al. (2003)).

Final Points

- Relatively strong evidence exists for dilute accretion flows in astrophysics (accretion onto supermassive black holes).
- The classic model of accretion is modified primarily in two ways: relative lack of radiation (requiring one to balance energetics properly); and lack of collisions (large anisotropic viscosities and thermal conductivities modify plasma stability).
- From a fluid approach, one observes that viscosity (through the MVI) and thermal conductivity (through the MTI) can destabilize the plasma.
- Fluid approach is limited to, at best, mildly collisional plasmas; a more careful and perhaps astrophysically appropriate approach involves a fluid-kinetic analysis.

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