The Magnetothermal Instability (MTI) and Its Role in the Transport of Angular Momentum in Hot, Dilute, Magnetized Accretion

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Abstract and Organization

Recent observations have demonstrated the prevalence of underluminous accretion flows in massive and supermassive central galactic black holes, for which the best studied example is that of Sagittarius A* at the center of our Milky Way. In addition, circular polarization measurements of millimeter-wavelength radiation from Sagittarius A* has shown the existence of measurable magnetic fields in the source. These flows are characterized by the radiatively inefficient accretion of a hot, mildly collisional to highly collisionless, and optically thin plasma onto a black hole. The energy generated through the accretion of matter down a gravitational down a gravitational well cannot be radiated and therefore must be transported outwards or locally dissipated. We show that collisionless and mildly collisional MTI, a MHD mode of a dilute rotationally supported plasma, can destabilize these magnetized, radiatively inefficient flows and can carry out angular momentum and can carry out angular momentum and energy in order to allow accretion to occur.

- First, we demonstrate the evidence for hot underluminous accretion onto black holes in which magnetic fields can play a role in modifying the local transport of angular momentum and energy. This is denoted as the **BACKGROUND**.
- Second, we construct a physical framework for the study of dilute rotationally supported MHD plasmas. We demonstrate evolution equations in the fluid regime (applicable to the outer regions of these flows) and in the kinetic MHD regime (applicable in the inner highly collisionless regions). We demonstrate quadratic correlations between fluid quantities that correspond to the fluxes of angular momentum and heat in dilute rotationally supported flows. This is denoted as the **PRELIMINARIES**.
- Third, we consider the instability of an equilibrium hot and dilute disk to the following physical effects:
  - In the fluid regime, the magnetoviscous-thermal instability (MVTI) – where both anisotropic viscous stresses and heat fluxes are dynamically important, and
  - In the kinetic regime, the analogue to the MVTI is denoted as the collisionless MTI.

  We describe the main features of both, as well as demonstrate that these instabilities give the right sign of accretion torque and heat flux to drive accretion in radiatively inefficient flows. This is denoted as **RESULTS**.
- Fourt, we discuss further numerical research of collisionless MHD phenomena in these astrophysical flows. This is denoted as **FURTHER RESEARCH**.

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Nomenclature

- s refers to the species of particle, whether ion or electron.
- "0" subscript denotes equilibrium quantity. δ prefix denotes perturbed quantity. Therefore, ρ_0 denotes equilibrium density and $\delta\rho$ the perturbed density. Furthermore, p_{s0} refers to the equilibrium pressure of species "s."
- $\theta = k_B\left(T_i + T_e\right)/\left(m_p + m_e\right)$ is the normalized thermal energy per unit mass, where T_i and T_e are ion and electron temperatures, respectively, and m_p and m_e are the ion (proton) and electron masses, respectively.
- $v_A = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén speed, where B_0 is the equilibrium magnetic field strength.
  $\beta = \theta_0/v_A^2$ is the ratio of isothermal sound to Alfvén speed, squared. This differs from the literature plasma β by a factor of 2.

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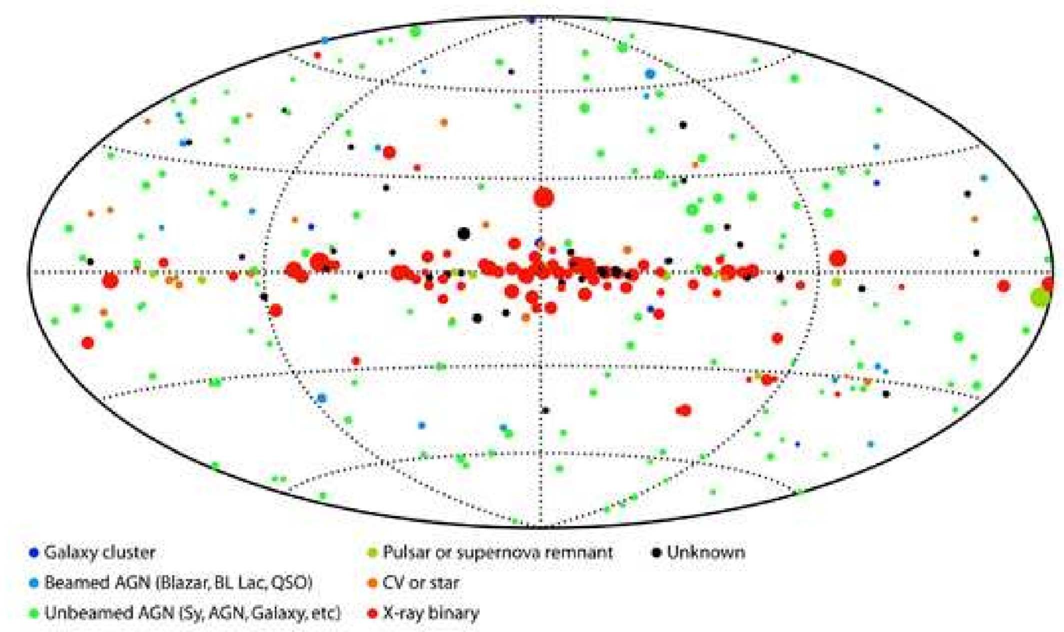
BACKGROUND

Commonality of RIAFs?

Only 200 AGNs or quasars (high-luminosity, 10% energy efficiency accretion onto black holes) within the nearest 4×10^8 ly according to recent AGN census; BUT extreme commonality of central massive ($M \gtrsim 10^6 M_\odot$) and supermassive ($M \gtrsim 10^8 M_\odot$) black holes [1], implying underluminous accretion is common.



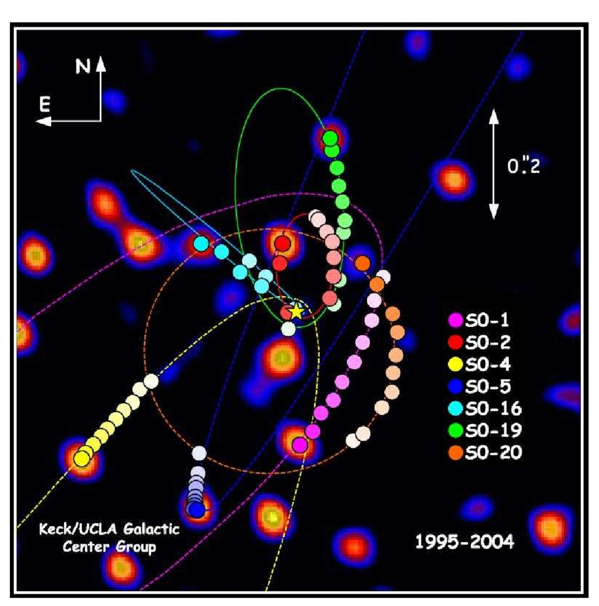
Evidence of underluminous accretion from Chandra X-ray observations of other nearby galaxies

Galaxy	d (Mpc)	$M_{\rm BH}$ ($\times 10^8 M_\odot$)	$R_{\rm Bondi}$ (arcsec)	$L_{\rm Bondi}$ (erg s^{-1})	L_X (erg s^{-1})
NGC 1399[a]	20.5	10.6	0.36	2.3×10^{44}	$\lesssim 9.7 \times 10^{38}$
NGC 4472[a]	16.7	5.65	0.24	4.5×10^{43}	$\lesssim 6.4 \times 10^{38}$
NGC 4636[a]	15.0	0.791	0.049	4.5×10^{41}	$\lesssim 2.7 \times 10^{38}$
M 82[b]	18.4	30	2	5×10^{44}	$\sim 7 \times 10^{40}$
Sag. A*[c]	8.5×10^{-3}	0.026[d]	2.2	6×10^{40}	2.2×10^{33}

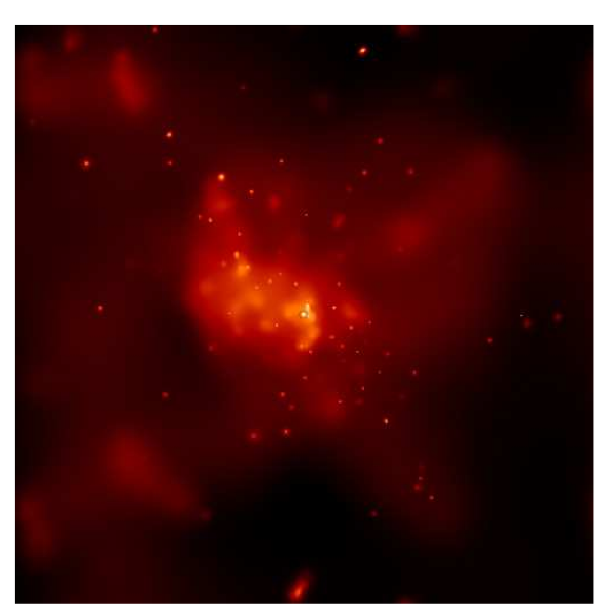
[a]Taken from [2]
[b]Taken from [3]
[c]Taken from [4]
[d]Mass measurement of Sag. A* taken from [5, 6]

Sagittarius A*

- ambient conditions from Chandra X-ray data [4] imply luminosity assuming efficient accretion from gravitational capture of $L_{\rm capture} \sim 6 \times 10^{40}$ erg s^{-1}.
- bolometric luminosity, primarily in the far IR and mm, of Sag. A* $L \sim 6 \times 10^{36}$ erg s$^{-1} \ll L_{\rm capture}$ [7, 8].



Time resolved orbits of stars orbiting Sagittarius A*, taken from `http://www.astro.virginia.edu/~ghezgroup/gc`.



False color Chandra X-ray image of the 2-10 keV emission within 2 pc of the central galactic black hole Sagittarius A*. The diffuse emission is attributed mainly to local shock and supernova heating, while point sources are associated with compact stellar emission. Image source is `http://chandra.harvard.edu/photo`.

Dilute Nature of Accretion

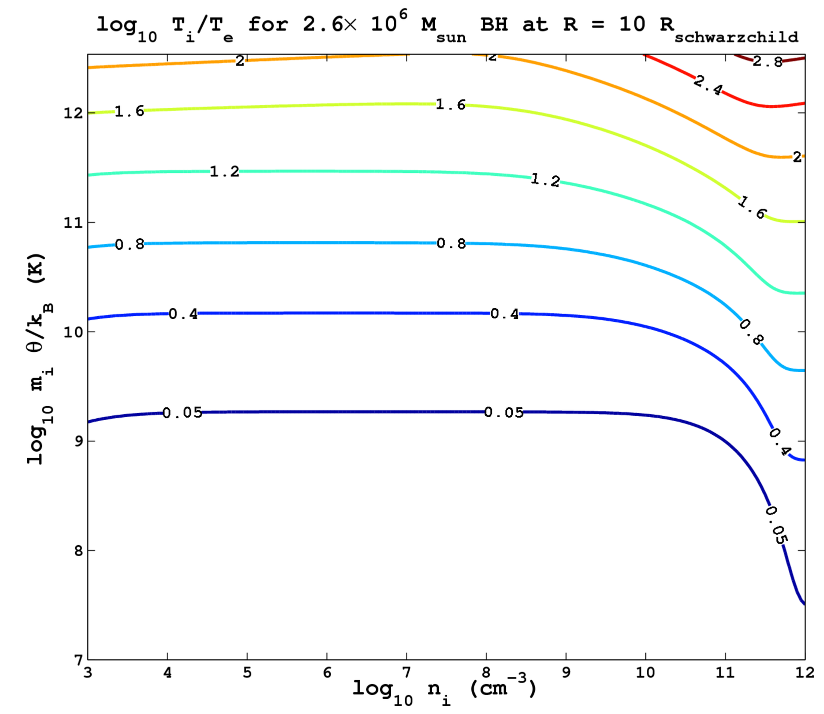
- Mildly collisional at best. The following table borrowed from [9] and other sources demonstrated this, with Chandra observations of the outer 1".

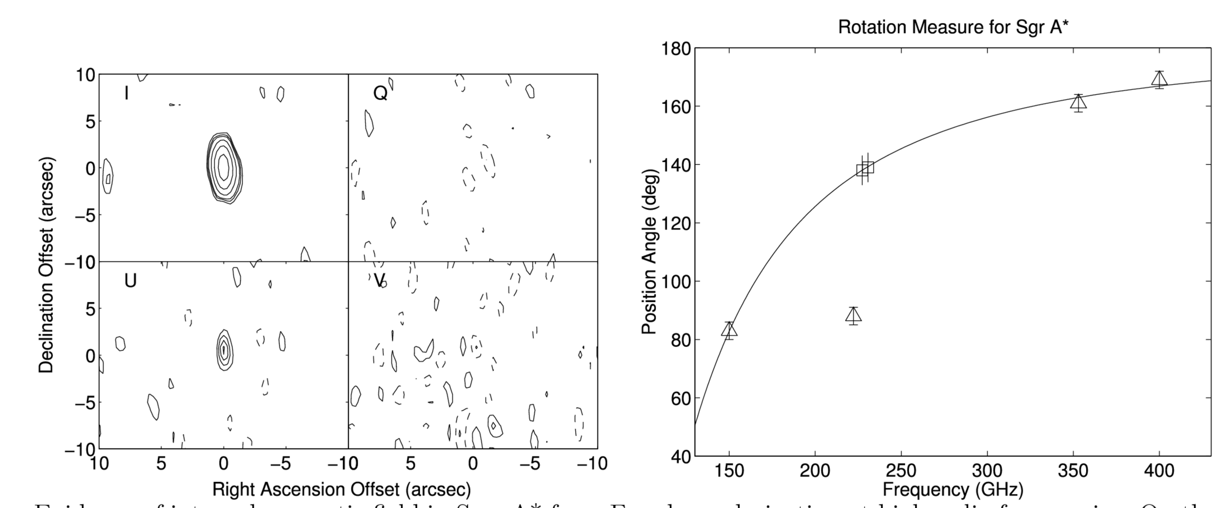
Galaxy	n(1") (cm^{-3})	T(1") (10^7 K)	R(1") (cm)	λ(1")/R(1")	λ(1")/$R_{\rm Bondi}$
Sag. A*[a]	100	2.3	1.3×10^{17}	0.4	0.4
NGC 1399[b]	0.3	0.9	3.1×10^{20}	0.009	0.02
NGC 4472[b]	0.2	0.9	2.5×10^{20}	0.016	0.07
NGC 4636[b]	0.07	0.7	2.2×10^{20}	0.032	0.6
M 82[c]	0.17	0.9	2.7×10^{20}	0.018	0.02
M 32[d]	0.07	0.4	1.2×10^{19}	0.2	1.3

[a]Taken from [4]
[b]Taken from [4]
[c]Taken from [3]
[d]Taken from [10]

Inner regions become highly collisionless.

- Radiatively inefficient and optically thin – very little energy generated by gravitational infall is radiated away, and the flow remains transparent to radiation.
- Two-temperature – ion-electron coupling is weak enough that ions may reach temperatures $\sim 10^{12}$ K in the inner regions, while electron maximum temperatures $T \sim 10^9 - 10^{10}$ K.





Evidence of internal magnetic field in Sag. A* from Faraday polarization at high radio frequencies. On the left is the unresolved 1" millimeter and far infrared radio emission from the Sag. A* sources. On the right is difference between left and right circularly polarized radiation from Sag. A* as a function of frequency; the best-fit rotation measure $RM = 4.3 \pm 0.1 \times 10^5$ rad m^{-2}. This plot is taken from [11].

PRELIMINARIES

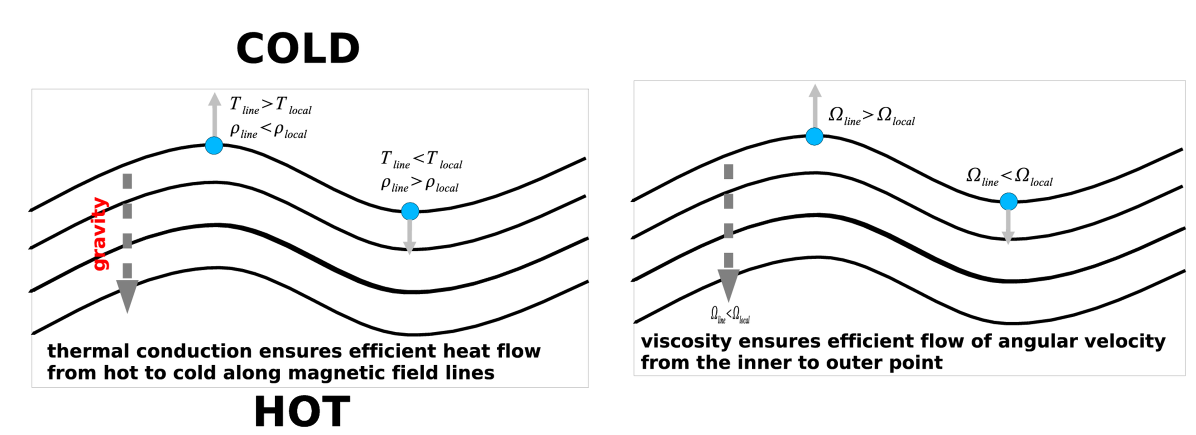
New MHD Instabilities

In a dilute plasma, even a weak magnetic field (with negligible Lorentz forces) may easily be swtrong enough that the ion gyrofrequency > collision frequency or other MHD dynamic timescale. In this case,

- heat flux and viscosity:

$$\text{heat flux:} \quad \mathbf{q} = q\mathbf{b}$$
$$\text{viscous stress tensor:} \quad \boldsymbol{\sigma} = \sigma_{\mathbf{bb}}\left(\mathbf{bb} - \frac{1}{3}\mathbb{I}\right)$$

- In the MVI, anisotropic viscous stresses destabilize plasmas with angular velocities that decrease outwards [12].
- In the MTI, anisotropic heat fluxes destabilize plasmas with adverse temperature (rather than entropy) profiles [13].



Equilibrium Disk Profile

- Plasma dynamics is considered in a rotating, cylindrical frame centered about the origin, with local velocity $\mathbf{u} = \mathbf{V} - R\Omega(R)\hat{\boldsymbol{\phi}}$.
- Equilibrium velocity $R\Omega(R)\hat{\boldsymbol{\phi}}$ approximately satisfies the equation,

$$\rho_0\Omega(R)R^2 \approx \frac{GM\rho_0}{R^3}$$

  With radial flow $|v_R| \ll R\Omega$ and isothermal sound speed $\sqrt{P_0/\rho_0} \ll R\Omega(R)$ (thin disk).
- Equilibrium nonradial magnetic field,

$$\mathbf{B}_0 = B_0\left(\hat{\boldsymbol{\phi}}\cos\chi + \hat{\boldsymbol{z}}\sin\chi\right).$$

- Isotherms along magnetic field lines (high thermal conductivity along magnetic fields), $T_0 \equiv T_0(R)$.
- Orbital angular velocity constant along surfaces of constant radius, $\Omega \equiv \Omega(R)$.
- Equilibrium profiles of temperature and pressure with the following normalized gradients,

$$\alpha_T = -H\frac{\partial \ln T_0}{\partial R}, \quad \alpha_P = -H\frac{\partial \ln p_0}{\partial R}, \quad H = \sqrt{\frac{p_0/\rho_0}{GM/R^3}}.$$

MHD Equations

Constituent MHD equations of continuity, force balance, thermal energy, and magnetic induction in a dilute plasma, where $\mathbf{u} = \mathbf{V} - R\Omega(R)\hat{\boldsymbol{\phi}}$:

$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\rho + \nabla\cdot(\rho\mathbf{u}) = 0,$$
$$\rho\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\mathbf{u} + 2\Omega\hat{\boldsymbol{z}}\times\rho\mathbf{u} + \Omega' R\rho u_R\hat{\boldsymbol{\phi}} = -\nabla\left(\frac{B^2}{8\pi}\right) + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi} + \nabla\cdot\left(\sigma_{\mathbf{bb}}\left[\mathbf{bb} - \frac{1}{3}\mathbb{I}\right]\right) + \frac{\rho}{\rho_0}\nabla p_0,$$
$$\frac{3}{2}\left(\left[\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right]p + \nabla\cdot[p\mathbf{u}]\right) + p\nabla\cdot\mathbf{u} = \nabla\cdot(q\mathbf{b}) + \sigma_{\mathbf{bb}}\left(\mathbf{b}\cdot\nabla\mathbf{u}\cdot\mathbf{b} - \frac{1}{3}\nabla\cdot\mathbf{u} + \Omega' R b_R b_\phi\right),$$
$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\mathbf{B} + \mathbf{u}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\left(\nabla\cdot\mathbf{u}\right) + \Omega' R B_R\hat{\boldsymbol{\phi}}.$$

- Fluid: the viscous stress and parallel heat flux are given by the following, where η_e and η_ν are (electron) thermal and (ion) viscous diffusivities whose values are given by [14],

$$q = \frac{5}{2}\eta_e\mathbf{b}\cdot\left(k_B T_e/m_e\right), \quad \sigma_{\mathbf{bb}} = 3\rho\eta_\nu\left(\mathbf{b}\cdot\nabla\mathbf{u} - \frac{1}{3}\nabla\cdot\mathbf{u} + \Omega' R b_R b_\phi\right).$$

- Kinetic: Parallel and perpendicular pressures to the magnetic field, $p_\parallel$ and $p_\perp$,

$$p = \frac{2}{3}p_\perp + \frac{1}{3}p_\parallel, \qquad \sigma_{\mathbf{bb}} = p_\parallel - p_\perp,$$
$$p_{s\parallel} = 2\pi m_s\int f_s v_\parallel^2\, dv_\parallel\, v_\perp\, dv_\perp, \quad p_{s\perp} = 2\pi m_s\int f_s\left(\frac{1}{2}v_\perp^2\right)dv_\parallel\, v_\perp\, dv_\perp.$$

  Pressures are calculated from reduced Boltzmann equation [15, 16] in rotating frame:

$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)f_s + \left(v_\parallel\mathbf{b} + \mathbf{u}_\perp\right)\cdot\nabla f_s + \left(\frac{Z_s eE_\parallel}{m_s} + \frac{m_p}{\rho_0 m_s}\mathbf{b}\cdot\nabla p_0\right)\frac{\partial f_s}{\partial v_\parallel} +$$
$$\left(-\mathbf{b}\cdot\left(\left[\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right]\mathbf{u}_\perp + \left[\mathbf{u}_\perp + v_\parallel\mathbf{b}\right]\cdot\nabla\mathbf{u}_\perp\right) + \frac{1}{2}v_\perp^2\nabla\cdot\mathbf{b}\right)\frac{\partial f_s}{\partial v_\parallel} +$$
$$2\Omega\hat{\boldsymbol{z}}\cdot\left(\mathbf{b}\times\mathbf{u}\right) - \Omega' R b_\phi\hat{\boldsymbol{R}}\cdot\left(\mathbf{u}_\perp + v_\parallel\mathbf{b}\right)\frac{\partial f_s}{\partial v_\parallel} = C\left\langle f - s\right\rangle.$$

  - $\mathbf{u}_\perp = \mathbf{u} - (\mathbf{u}\cdot\mathbf{b})\,\mathbf{b}$ is the MHD flow velocity perpendicular to the magnetic field; $E_\parallel = \mathbf{E}\cdot\mathbf{b}$ is the (parallel) electric field that ensures quasineutrality, $n_i = n_e$.
  - $v_\parallel$ and $v_\perp$ are velocities of the particle distribution function $f_s\left(v_\parallel, v_\perp, \mathbf{x}, t\right)$, about the equilibrium flow, perpendicular and parallel to the magnetic field.

  Terms in red are mirror forces (arising from conservation of magnetic moment in particle distribution function); terms in dark blue are noninertial (Coriolis and tidal) forces arising from comoving with a differentially rotating fluid; terms in dark green are forces arising from equilibrium gradients; and $C\left\langle f_s\right\rangle$ is a simplified collision operator appropriate in the absence of sharp velocity gradients in the distribution function [17].

Mechanics of Accretion

Necessary conditions to get accretion in *radiatively inefficient* dilute rotationally supported flows,

- Accretion requires angular momentum to be transported outwards, and energy generated from gravitational infall to be transported or radiated:

$$2\pi R^2\left\langle T_{R\phi}\right\rangle + R\Omega(R)\dot{M} = R_{\rm in}\Omega\left(R_{\rm in}\right)\dot{M}$$
$$\frac{1}{R}\frac{\partial}{\partial R}R\left\langle q_R\right\rangle - \frac{\dot{M}}{2\pi R^2}s\left(\frac{1}{\rho_0}\frac{\partial p_0}{\partial R}\right) = -\frac{\partial\Omega}{\partial\ln R}\left\langle T_{R\phi}\right\rangle$$

- **Angular momentum flux**: $T_{R\phi} = \left\langle\rho_0\delta u_R\delta u_\phi - \frac{\delta B_R\delta B_\phi}{4\pi} + \delta\sigma_{\mathbf{bb}}\delta b_R\cos\chi\right\rangle > 0.$
- **Radial heat flux**: $q_R = \left\langle\frac{5}{2}\rho_0\delta\theta\delta u_R + \delta q\delta b_R - \frac{1}{3}\delta\sigma_{\mathbf{bb}}\delta u_R\right\rangle > 0.$
- $\langle\delta a\delta b\rangle$ is a spatially averaged correlation of fluctuations. Quadratic fluctuations dominate because small fluctuations, with relatively weak magnetic fields, are likely to occur.
- Heat flux condition first noted in [18], and modifications to the angular momentum flux was also noted in [19]. We estimate these for our unstable modes as the following,

$$T_{R\phi} = \mathrm{Re}\left(\rho_0\delta u_R\delta u_\phi^* - \frac{\delta B_R\delta B_\phi^*}{4\pi} + \delta\sigma_{\mathbf{bb}}\delta b_R^*\cos\chi\right) > 0,$$
$$q_R = \mathrm{Re}\left(\frac{5}{2}\rho_0\delta\theta\delta u_R^* + \delta q\delta b_R^* - \frac{1}{3}\delta\sigma_{\mathbf{bb}}\delta b_R^*\right).$$