The Magnetothermal Instability in Dilute Accreting Plasmas

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Introduction

Recent observations have demonstrated the prevalence of underluminous accretion flows in massive and supermassive central galactic black holes, for which the best studied example is that of Sagittarius A* at the center of our Milky Way. These flows are characterized by the radiatively inefficient accretion of a hot, mildly collisional to highly collisionless, and optically thin plasma onto a black hole. In these plasmas, even an extremely weak magnetic field can lead to anisotropic heat fluxes and viscous stresses directed along field lines. This allows for the development of plasmas unstable to rotational shear via anisotropic viscous stresses (the magnetoviscous instability, or MVI [1]), and to adverse temperature (rather than entropy) gradients via anisotropic heat fluxes (the magnetothermal instability, or MTI [2]). Furthermore, in these radiatively inefficient and slender rotationally supported flows, the energy generated through gravitational infall must be advected through local thermal turbulence.

In this poster we justify our research astrophysically. We explain some of the salient features of hot underluminous accretion onto black holes, and describe physical models of the MVI and MTI that are expected to operate in these dilute flows. We demonstrate the salient evolution equations that describe the MHD dynamics of a dilute plasma, in both the fluid regime (applicable in the outer regions of these flows) and in the kinetic MHD regime (applicable in the inner highly collisionless regions). We consider an equilibrium hot and dilute disk, and consider the instability of this disk to the following instabilities:

- In the fluid regime, the magnetoviscous-thermal instability (MVTI) – where both anisotropic viscous stresses and heat fluxes are dynamically important, and
- In the kinetic regime, the analogue to the MVTI is denoted denote as the collisionless MTI.

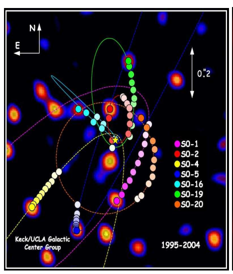
We describe salient features of both, as well as demonstrate that these instabilities give the right sign of accretion torques and heat flux to drive accretion in radiatively inefficient flows. We end with conclusions and possible effects of the nonlinear MVTI and collisionless MTI in astrophysical objects.

Evidence for Underluminous Accretion

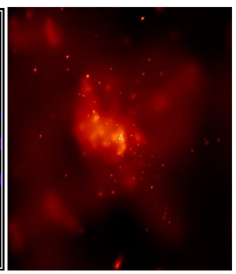
- Only 200 AGNs or quasars (high-luminosity, 10% mass energy efficiency accretion onto black holes) within the nearest 4×10^8 ly according to recent AGN census; BUT extreme commonality of central massive ($M \gtrsim 10^5 M_\odot$) and supermassive ($M \gtrsim 10^8 M_\odot$) black holes [3], implying underluminous accretion is common.
- Best evidence: Sagittarius A* black hole at the center of our Milky Way.
  - $M \sim 2.6 \times 10^6 M_\odot$ from time-lapse stellar orbits [4, 5].
  - ambient conditions from Chandra X-ray data [6] imply luminosity assuming efficient accretion from gravitational capture of $L_{\text{capture}} \sim 6 \times 10^{40}$ erg s^{-1}.
  - bolometric luminosity, primarily in the far IR and mm, of Sag. A* $L \sim 6 \times 10^{36}$ erg s$^{-1} \ll L_{\text{Bondi}}$ [7, 8].
- Evidence of underluminous accretion from Chandra X-ray observations of other nearby galaxies:

Galaxy	d (Mpc)	M_{BH} ($\times 10^8 M_\odot$)	R_{Bondi} (arcsec)	L_{Bondi} (erg s^{-1})	L_X (erg s^{-1})
NGC 1399[a]	20.5	10.6	0.36	2.3×10^{44}	$\lesssim 9.7 \times 10^{38}$
NGC 4472[a]	16.7	5.65	0.24	4.5×10^{43}	$\lesssim 6.4 \times 10^{38}$
NGC 4636[a]	15.0	0.791	0.049	4.5×10^{41}	$\lesssim 2.7 \times 10^{38}$
M 82[b]	18.4	30	2	5×10^{44}	$\sim 7 \times 10^{40}$
Sag. A*[c]	8.5×10^{-3}	0.026[d]	2.2	6×10^{40}	2.2×10^{33}

[a] Taken from [9]
[b] Taken from [10]
[c] Taken from [6]
[d] Mass measurement taken from [4, 5]



Time resolved orbits of stars orbiting Sagittarius A*, taken from http://www.astro.ucla.edu/ ghezgroup/gc



False color Chandra X ray image of 2-10 keV emission within 2 pc of the central galactic black hole Sagittarius A*. The diffuse emission is attributed mainly to local shock heating and supernova heating, while point sources are associated with compact stellar emission. Image source is http://chandra.harvard.edu/photo

Properties of Underluminous Accretion

- **Mildly collisional** at best. The following table borrowed from [11] and other sources demonstrates this, with Chandra observations at the outer 1".

Galaxy	$n\,(1")$ (cm^{-3})	$T\,(1")$ (10^7 K)	$R\,(1")$ (cm)	$\lambda\,(1")/R\,(1")$	$\lambda\,(1")/R_{\text{Bondi}}$
Sag. A*[a]	100	2.3	1.3×10^{17}	0.4	0.4
NGC 1399[b]	0.3	0.9	3.1×10^{20}	0.009	0.02
NGC 4472[b]	0.2	0.9	2.5×10^{20}	0.016	0.07
NGC 4636[b]	0.07	0.7	2.2×10^{20}	0.032	0.6
M 82[c]	0.17	0.9	2.7×10^{20}	0.018	0.02
M 32[d]	0.07	0.4	1.2×10^{19}	0.2	1.3

[a] Taken from [6]
[b] Taken from [9]
[c] Taken from [10]
[d] Taken from [12]

Inner regions become highly collisionless.

- **Radiatively inefficient and optically thin** – very little energy generated by gravitational infall is radiated away, and the flow remains transparent to radiation.
- **Two-temperature** – ion-electron coupling is weak enough that ions may reach temperatures $\sim 10^{12}$ K in the inner regions, while electron maximum temperatures $T \sim 10^9 - 10^{10}$ K.

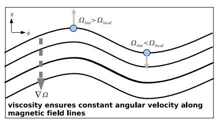
Dilute Plasma Transport and Additional Instabilities

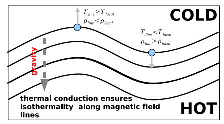
In a dilute plasma, even a weak magnetic field (with negligible Lorentz forces) may easily be strong enough that the ion gyrofrequency > collisional frequency.

- Heat flux and viscosity:

$$\text{heat flux} : \mathbf{q} = q\mathbf{b},$$
$$\text{viscosity} : \boldsymbol{\sigma} = \sigma_{\mathbf{bb}}\left(\mathbf{bb} - \frac{1}{3}\mathbb{I}\right)$$

- In the MVI, anisotropic viscosity destabilizes plasmas with angular velocity gradients [1]
- In the MTI, anisotropic heat flux destabilizes plasmas with adverse temperature (rather than entropy) gradients [2].





Equilibrium Disk Profile

- Plasma dynamics is considered in a rotating, cylindrical frame centered about the central mass.
  - Local velocity $\mathbf{u} = \mathbf{V} - R\Omega(R)\hat{\boldsymbol{\phi}}$.
  - p: total pressure; ρ: mass density.
  - $\mathbf{B}$: magnetic field; B: magnetic field magnitude.
- Equilibrium velocity $\mathbf{V}_0 \equiv R\Omega(R)\hat{\boldsymbol{\phi}}$ with radial force balance:

$$\rho_0\Omega^2 R = \frac{\partial p_0}{\partial R} + \frac{GM\rho_0}{R^3}.$$

With radial inflow velocities $|v_R| \ll R\Omega$.

- Equilibrium nonradial magnetic fields:

$$\mathbf{B}_0 = B_0\left(\hat{\boldsymbol{\phi}}\cos\chi + \hat{\boldsymbol{z}}\sin\chi\right).$$

- Isotherms along magnetic field lines (high thermal conductivity along magnetic fields), $T_0 \equiv T_0\,(R)$.
- Equilibrium profiles with radial

$$\alpha_T = -H\frac{\partial \ln T_0}{\partial R} > 1, \alpha_P = -H\frac{\partial \ln P_0}{\partial R} > 1, H = \sqrt{\frac{P_0/\rho_0}{GM/R^3}}.$$

Constituent Equations

Constituent MHD equations in a dilute plasma, where $\mathbf{u} = \mathbf{V} - R\Omega(R)\hat{\boldsymbol{\phi}}$:

$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\rho + \nabla\cdot(\rho\mathbf{u}) = 0,$$
$$\rho\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\mathbf{u} + 2\Omega\hat{\mathbf{z}}\times\rho\mathbf{u} + \Omega' R\rho u_R\hat{\boldsymbol{\phi}} = -\nabla\left(\frac{B^2}{8\pi}\right) + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi} + \nabla\cdot\left(\sigma_{\mathbf{bb}}\left[\mathbf{bb} - \frac{1}{3}\mathbb{I}\right]\right) + \frac{\rho}{\rho_0}\nabla p_0,$$
$$\frac{3}{2}\left(\left[\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right]p + \nabla\cdot(p\mathbf{u})\right) + p\nabla\cdot\mathbf{u} = \nabla\cdot(q\mathbf{b}) + \sigma_{\mathbf{bb}}\left(\mathbf{b}\cdot\nabla\mathbf{u}\cdot\mathbf{b} - \frac{1}{3}\nabla\cdot\mathbf{u} + \Omega' R b_R b_\phi\right),$$
$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)\mathbf{B} + \mathbf{u}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}(\nabla\cdot\mathbf{u}) + \Omega' R B_R\hat{\boldsymbol{\phi}}.$$

- **Fluid**: the viscous stress and parallel heat flux are given by the following, where η_e and η_i are (electron) thermal and (ion) viscous diffusivities whose values are given by [13]

$$q = \frac{3}{2}\eta_e\mathbf{b}\cdot\nabla(k_B T_e/m_e), \quad \sigma_{\mathbf{bb}} = 3\rho\eta_i\left(\mathbf{b}\cdot\nabla\mathbf{u}\cdot\mathbf{b} - \frac{1}{3}\nabla\cdot\mathbf{u} + \Omega' R b_R b_\phi\right)$$

The energy balance equation:

- **Kinetic**: parallel and perpendicular pressures to the magnetic field field $\delta p_\parallel$ and $\delta p_\perp$.

$$p = \frac{2}{3}p_\perp + \frac{1}{3}p_\parallel, \quad \sigma_{\mathbf{bb}} = p_\parallel - p_\perp.$$

Pressures calculated from a reduced Boltzmann equation [14] in rotating frame:

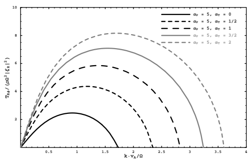
$$\left(\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right)f_s + (v_\parallel\mathbf{b} + \mathbf{u}_\perp)\cdot\nabla f_s + \left(\frac{Z_s e E_\parallel}{m_s} + \frac{m_i}{\rho_0 m_s}\mathbf{b}\cdot\nabla p_0\right)\frac{\partial f_s}{\partial v_\parallel} + \left(-\mathbf{b}\cdot\left(\left[\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial\phi}\right]\mathbf{u}_\perp + [\mathbf{u}_\perp + v_\parallel\mathbf{b}]\cdot\nabla\mathbf{u}_\perp\right) + \frac{1}{2}v_\perp^2\nabla\cdot\mathbf{b}\right)\frac{\partial f_s}{\partial v_\parallel} + \left(2\Omega\hat{\mathbf{z}}\cdot(\mathbf{b}\times\mathbf{u}) - \Omega' R b_\phi \hat{\boldsymbol{R}}\cdot(\mathbf{u}_\perp + v_\parallel\mathbf{b})\right)\frac{\partial f_s}{\partial v_\parallel} = C[f_s].$$

- s refers to ions or electrons, (i) for ions, (e) for electrons.
- $\mathbf{u}_\perp = \mathbf{u} - \mathbf{b}(\mathbf{u}\cdot\mathbf{b})$ is MHD flow velocity perpendicular to the magnetic field; $E_\parallel = \mathbf{E}\cdot\mathbf{b}$ is the (parallel) electric field that ensures quasineutrality, $n_i = n_e$.
- $v_\parallel$ and $v_\perp$ are velocities of particle distribution function $f_s(v_\parallel, v_\perp, \mathbf{x}, t)$, about equilibrium flow, perpendicular and parallel to magnetic field.

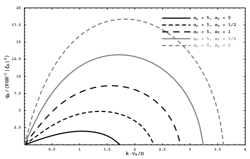
Term in red is mirror force, blue are noninertial forces arising from a differentially rotating fluid (Coriolis and tidal forces), dark green are forces arising from equilibrium gradients, and $C[f_s]$ is a very simple collision operator [15].

$$p_{\parallel s} = 2\pi m_s\int f_s v_\parallel^2\, dv_\parallel v_\perp\, dv_\perp, \quad p_{\perp s} = 2\pi m_s\int f_s\left(\frac{1}{2}v_\perp^2\right) dv_\parallel v_\perp\, dv_\perp$$

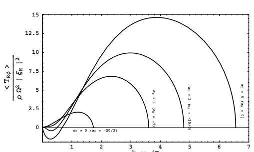
Quadratic Fluxes



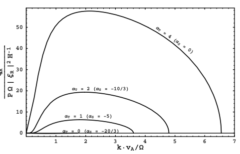
collisionless MTI azimuthal stress



collisionless MTI heat flux



MVTI azimuthal stress



MVTI heat flux

outward quadratic heat flux and azimuthal stress (angular momentum torque) in both collisional and collisionless regimes

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Model Equations and Quadratic Correlations

Modal analysis of the collisionless MTI/MVTI:

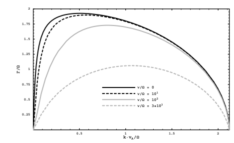
- Axisymmetric perturbations $\delta A \propto \exp(\Gamma t + ik_Z z)$, Γ the growth rate, k_Z the vertical wavenumber.
- Both **fluid** and **kinetic** equilibria have a single temperature as a simplification.
- **fluid** approach applicable in outer disk: fixed Prandtl number $\text{Pr} = \eta_i/\eta_e \approx 1/101$ [13].

Necessary conditions to get accretion (inward net mass flux) in *radiatively inefficient* flows, with our problem setup

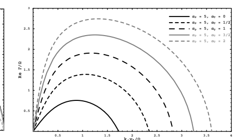
- **Azimuthal stress**: $T_{R\phi} = \left\langle \rho_0\delta u_R\delta u_\phi - \frac{\delta B_R\delta B_\phi}{4\pi} + \delta\sigma_{\mathbf{bb}}\delta b_R\cos\chi\right\rangle > 0.$
- **heat flux**: $q_R = \left\langle\frac{5\rho_0 k_B}{2m_i}(\delta T_i + \delta T_e)\delta u_R + \delta q\delta b_R - \frac{1}{3}\delta\sigma_{\mathbf{bb}}\delta u_R\right\rangle > 0.$
- Where $\langle\delta a\delta b\rangle$ is a spatially-averaged quadratic correlation of fluctuations.
- Heat flux condition first noted in [16], and modifications to azimuthal stress (quadratic torque) was also noted in [17]. We estimate as following:

$$T_{R\phi} \equiv \text{Re}\left(\rho\delta u_R\delta u_\phi^* - \frac{\delta B_R\delta B_\phi^*}{4\pi} + \delta\sigma_{\mathbf{bb}}\delta b_R^*\cos\chi\right) > 0,$$
$$q_R \equiv \text{Re}\left(\frac{5\rho_0 k_B}{m_i}(\delta T_i + \delta T_e)\delta u_R^* + \delta q\delta b_R^* - \frac{1}{3}\delta\sigma_{\mathbf{bb}}\delta u_R^*\right).$$

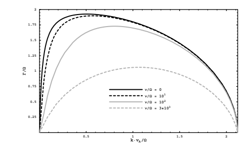
Dispersion Relation



MVTI with small viscosity



collisionless MTI, β = 100

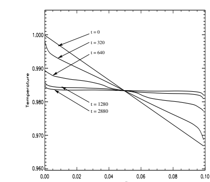


collisional MTI to MVTI, β = 100

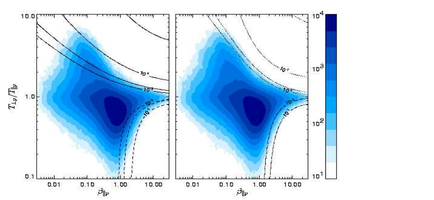
- For collisionless plasmas, damping long wavelength modes with phase velocities of the order sound speed $k_\parallel \lesssim \Omega\,(P_0/\rho_0)^{1/2}$.
- For weakly magnetized plasmas, reducing to the fluid approximation for $\nu \gtrsim \Omega\beta$ – i.e., when wavelength of fastest growing modes $\lesssim \lambda_i$, ion collisional mean free path [18].

Further Research

- Nonlinear development of the collisionless MTI (or MVTI), from [19], may imply a disk flow whose "averaged" energetics is dominated by *radial thermal conductivity*. A first attempt to consider a large phenomenological viscosity has been done by [11, 20].
- Pressure anisotropy easily excited in collisionless plasmas, may lead to an astrophysical flow in which the viscous pressure $|p_\parallel - p_\perp| \lesssim p/\beta$ threshold unstable to gyrokinetic firehose/mirror instabilities (see, e.g., [21], for evidence of solar wind threshold instability).
- Global simulations are **necessary** for descriptions of radiatively inefficient flows; codes that explicitly conserve all energy, such as Athena [22], are necessary to model the nonlinear rotational MTI.



Evolution of the temperature profile [19], with only an anisotropic heat flux, in a magnetized fluid in which the bottom is kept hot and the top is kept cool. The bulk of the fluid quickly reaches isothermality due to the MTI.



slow solar wind plasma kept threshold unstable to "fast" gyrokinetic instabilities