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Transport and Stability Analysis of Dilute Magnetized Accreting Flows

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3 December 2007



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Outline

Basic Theory of Classical Accretion

Classical Accretion Insufficiency of Hydrodynamics Magnetorotational Instability

Astrophysical Evidence of Underluminous Flow

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MHD Instabilities

The Magnetoviscous and Magnetothermal Instabilities Mechanics of Dilute Accretion Stability Analysis Equilibrium Disk and Stability Analysis The MVTI Collisionless MTI Further Work Numerical Work Summary

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| Classical Accre | etion | | | | | |
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Mechanics of Accretion

Angular momentum must be transported outwards:

$$2\pi R^2 \langle T_{R\phi} \rangle + R\Omega(R) \dot{M} = R_{\rm in} \Omega(R_{\rm in}) \dot{M}.$$

Where $\langle T_{R\phi} \rangle > 0$ is the averaged angular momentum flux carried out by the turbulence, \dot{M} is the mass inflow rate, and we impose a no-stress condition at the inner boundary.

 Energy generated through gravitational infall must be radiated or transported. A local model of energy transport (Balbus, 2004a):

$$\frac{1}{R}\frac{\partial}{\partial R}R\left\langle F_{ER}\right\rangle -\frac{\dot{M}}{2\pi R^{2}}\left(\frac{1}{\rho_{0}}\frac{\partial p_{0}}{\partial R}\right)=-Q_{-}-\left\langle T_{R\phi}\right\rangle \frac{\partial\Omega}{\partial\ln R}.$$

Where $\langle F_{ER} \rangle$ is a rate of local turbulent heat flux and Q_{-} is a radiative dissipation rate.

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| Classical Accre | etion | | | | | |

Classical Accretion Disk

- optically thick and/or radiatively efficient
 - $Q_{-} \approx \langle T_{R\phi} \rangle \frac{\partial \Omega}{\partial \ln R}$. Therefore $L_{\text{accretion}} \rightarrow GM\dot{M}/(2R_{\text{in}})$.
- Temperature in outer disk goes as $T_{\rm disk} \propto R^{-3/4}$. Assuming blackbody, disk spectrum (see, e.g., Lynden-Bell (1969); Frank et al. (2002)):





Fit of AGN "big blue bump" spectrum to classical disk blackbody. Figure is taken from Francis et al. (1991)

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| Insufficiency o | f Hydrodynamics | | | | | |

Insufficiency of Hydrodynamics

- Extremely high Reynolds numbers of astrophysical accretion flows led many to believe hydrodynamic turbulence as a source of angular momentum transport, but
- Hydrodynamic Keplerian flows are linearly stable.
 - numerical simulations (Balbus et al., 1996; Hawley et al., 1998; Lesur & Longaretti, 2005) demonstrate nonlinear stability and dissipation of initially random flows.
 - High-reynolds number hydrodynamic experiment (Ji et al., 2006) also demonstrates dynamically insignificant hydrodynamic turbulence and no angular momentum transport.

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Insufficiency of Hydrodynamics



Only those simulations in which there is pure shear or in which $q = \partial \ln \Omega / \partial \ln R > 2$ demonstrate kinetic turbulence sustained against numerical dissipation. This figure is taken from Balbus et al. (1996).



Angular momentum carried by the the Ji et al. (2006) Princeton Rayleigh-stable Taylor-Couette flow was found to be zero to within experimental errors for the highest Reynolds number trials.

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| Magnetorotati | onal Instability | | | | | |

The Magnetorotational Instability

- Velikhov (1959) and Chandrasekhar (1960) noted the linear instability of a metallic Taylor-Couette flow in which dΩ/dR < 0, in the presence of arbitrarily small magnetic fields.
- Balbus & Hawley (1991) noted its promise in driving MHD turbulence that can allow accretion in a wide class of astrophysical systems.
- Observed in local and global numerical simulations, and in beginning experimental simulations of Rayleigh-stable Taylor-Couette metallic flows.

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Magnetorotational Instability

Cartoon Model of the MRI



Tension provides a torque that speeds up the outer mass and slows down the inner mass. Angular momentum is transferred outwards, the spring tension increases, and the process runs away. Process ends when the tension is large enough that oscillation periods due to magnetic forces \gtrsim orbital period.

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| Magnetorotat | ional Instability | | | | | |

Local Nonlinear Simulations







On the left is the evolution of the magnetic contribution, and on the right the kinetic contribution, to $\langle T_{R\phi}\rangle$. One can observe a net outwards flux of angular momentum associated with the nonlinear development of the MRI. The time is in units of orbital period. These figures are taken from Hawley et al. (1996).

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Magnetorotational Instability

Global Nonlinear Simulation





Azimuthally averaged density of a long time snapshot of a global MRI simulation of accretion onto a black hole, taken from De Villiers et al. (2003). These global simulations have the following features:

- relatively slender disks (thermal velocities
 - $\stackrel{<}{_\sim}$ orbital velocities)
- outlying coronal region of dilute, magnetic pressure dominated plasma
- centrifugally evacuated funnel region about rotation axis.

Time averaged mass accretion and outflow rates in initial global simulations of the MRI (Hawley et al., 2001). Accretion rates are low, the mass accretion and outflow rates decreases with decreasing radius down to the marginally stable orbit.

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| Characteristics | s of RIAFs | | | | | |

Characteristics of RIAFs

- At the current epoch, classical or radiatively efficient accretion onto central galactic black holes is relatively rare.
- Likely that most forms of accretion onto central galactic black holes are RIAFs (radiatively inefficient accretion flows)
 - very little gravitational energy is radiated, and optically thin.
 - Two-temperature nature of plasma in primary emitting regions

 ions remain virial and adiabatic, electrons much cooler.
 - Dilute: ion mean free path of order or much larger than system scale.

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Commonality of RIAFs?



Complete survey of AGNs and quasars within the nearest 4 \times 10⁸ ly within Earth, taken with the SWIFT X-ray probe, taken from Winter et al. (2007). There exist only \sim 200 instances of high-energy accretion onto supermassive central galactic black holes. This is only a few percent of total number of central galactic black holes (Richstone et al., 1998).

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| Characteristics of RIAFs | | | | | | |

Underluminosity of RIAFs

Accretion Flow and X-Ray Luminosities of Dim Galactic Black Holes

| Galaxy | d (Mpc) | $M_{ m BH} \ (imes 10^8 M_{\odot})$ | R _{capture} (arcsec) | $ \begin{array}{c} L_{capture} \\ (erg \ s^{-1}) \end{array} $ | $\begin{array}{c} \mathcal{L}_{\mathcal{X}} \\ (\text{erg s}^{-1}) \end{array}$ |
|-----------------------|---------------------|--------------------------------------|----------------------------------|--|---|
| NGC 1399 ¹ | 20.5 | 10.6 | 0.36 | $2.3 	imes 10^{44}$ | $\stackrel{<}{_\sim}9.7	imes10^{38}$ |
| NGC 4472 ¹ | 16.7 | 5.65 | 0.24 | 4.5×10^{43} | $\stackrel{<}{_\sim} 6.4 	imes 10^{38}$ |
| NGC 4636 ¹ | 15.0 | 0.791 | 0.049 | $4.5 	imes 10^{41}$ | $\stackrel{<}{_\sim} 2.7 	imes 10^{38}$ |
| M 82 ² | 18.4 | 30 | 2 | $5	imes10^{44}$ | $\sim 7 	imes 10^{40}$ |
| Sag. A* ³ | $8.5 	imes 10^{-3}$ | 0.0264 | 2.2 | $6	imes 10^{40}$ | $2.2 	imes 10^{33}$ |

^aTaken from Loewenstein et al. (2001)

^bTaken from Di Matteo et al. (2003)

^cTaken from Baganoff et al. (2003)

^dMass measurement of Sag. A* taken from Schödel et al. (2002); Ghez et al. (2003)

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| Characteristics | of RIAEs | | | | | |

Dilute Nature of RIAF Plasma

Dilute Nature of Dim Accreting Galactic Black Holes (table taken from Menou (2005))

| Galaxy | $n(1")$ (cm^{-3}) | <i>T</i> (1") (10 ⁷ K) | R (1") (cm) | λ (1") /R (1") | λ (1") / $R_{capture}$ |
|-----------------------|---------------------|--------------------------------------|---------------------|------------------------|--------------------------------|
| Sag. A*1 | 100 | 2.3 | $1.3 	imes 10^{17}$ | 0.4 | 0.4 |
| NGC 1399 ² | 0.3 | 0.9 | $3.1	imes10^{20}$ | 0.009 | 0.02 |
| NGC 4472 ² | 0.2 | 0.9 | $2.5	imes10^{20}$ | 0.016 | 0.07 |
| NGC 4636 ² | 0.07 | 0.7 | $2.2	imes10^{20}$ | 0.032 | 0.6 |
| M 82 ³ | 0.17 | 0.9 | $2.7 	imes 10^{20}$ | 0.018 | 0.02 |
| M 32 ⁴ | 0.07 | 0.4 | $1.2 	imes 10^{19}$ | 0.2 | 1.3 |

^aTaken from Baganoff et al. (2003) ^bTaken from Baganoff et al. (2003) ^cTaken from Di Matteo et al. (2003) ^dTaken from Ho et al. (2003)

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| Best Example of RIAES: Sagittarius A* | | | | | | |

Sagittarius A*



from

False color Chandra X-ray image of the 2-10 keV emission within 2 pc of the central galactic black hole Sagittarius A*. Image source is http://chandra.harvard.edu/photo.

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Analysis of Dilute Astrophysical Flows

Sagittarius

http://www.astro.ucla.edu/~ghezgroup/gc.

A*.

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| Best Example | of RIAFS: Sagittarius A* | | | | | |

Extreme Underluminosity of Sagittarius A*

- ▶ ambient conditions from Chandra X-ray data (Baganoff et al., 2003) imply $L_{\text{capture}} \sim 6 \times 10^{40} \text{ erg s}^{-1}$.
- ▶ Bolometric luminosity L ~ 6 × 10³⁶ erg s⁻¹ ≪ L_{capture} (Beckert et al., 1996; Narayan et al., 1998).
- Spectrum of Sag. A* not a good fit for efficient accretion.



Poor spectral fits of Sag. A* bolometric emission to classical accretion disk spectrum at various mass accretion rates. This figure is taken from Narayan (2002).

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| Best Example of RIAFS: Savittarius A* | | | | | | |

Magnetic Fields About Sagittarius A*



Evidence of internal magnetic field in Sag. A* from Faraday polarization at high radio frequencies. On the left is the unresolved 1" millimeter and far infrared radio emission from the Sag. A* sources. On the right is difference between left and right circularly polarized radiation from Sag. A* as a function of frequency; the best-fit rotation measure $RM = 4.3 \pm 0.1 \times 10^5$ rad m⁻². This plot is taken from Bower et al. (2003).

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The Magnetoviscous and Magnetothermal Instabilities

In dilute plasmas in which ion cyclotron frequency \gg inverse time scales of our problem of interest (such as collision frequency), momentum and energy transport become anisotropic and may become dynamically important.

Heat flux and viscous stress tensor are modified:

heat flux:
$$\mathbf{q} = q\mathbf{b}$$

viscous stress tensor: $\boldsymbol{\sigma} = \sigma_{\mathbf{b}\mathbf{b}} \left(\mathbf{b}\mathbf{b} - \frac{1}{3}\mathbb{I}\right)$

- The magnetothermal instability (MTI) transports thermal energy via magnetic-field-directed thermal flux where temperature decreases upwards/outwards (Balbus, 2001).
- The magnetoviscous instability (MVI) transports angular momentum outwards, via magnetic-field directed viscous stresses, when angular velocity decreases outwards (Balbus, 2004b; Islam & Balbus, 2005).

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Magnetothermal Instability

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Magnetoviscous Instability



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| Mechanics of Dilute Accretion | | | | | | |

Mechanics of Accretion in Dilute RIAF Disks

For dilute, radiatively inefficient flows, turbulence must carry out both angular momentum and thermal energy:

$$2\pi R^{2} \langle T_{R\phi} \rangle + R\Omega(R)\dot{M} = R_{in}\Omega(R_{in})\dot{M}$$
$$\frac{1}{R}\frac{\partial}{\partial R}R \langle F_{ER} \rangle - \frac{\dot{M}}{2\pi R^{2}} \left(\frac{1}{\rho_{0}}\frac{\partial p_{0}}{\partial R}\right) = -\frac{\partial\Omega}{\partial\ln R} \langle T_{R\phi} \rangle$$

► Angular momentum flux: $\langle T_{R\phi} \rangle = \left\langle \rho_0 \delta u_R \delta u_\phi - \frac{\delta B_R \delta B_\phi}{4\pi} + \delta \sigma_{bb} \delta b_R b_{\phi 0} \right\rangle.$ ► Radial heat flux: $\langle F_{ER} \rangle = \left\langle \frac{5}{2} \rho_0 \delta \theta \delta u_R + \delta q \delta b_R - \frac{1}{3} \delta \sigma_{bb} \delta u_R \right\rangle.$

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| Equilibrium Disk and Stability Analysis | | | | | | |

Equilibrium Disk Structure

- Plasma dynamics is considered in a rotating, cylindrical frame centered about the origin.
- ► Thin disk: $\Omega(R)^2 \approx GM/R^3$; isothermal sound speed $\theta_0 = p_0/\rho_0 \ll R^2 \Omega(R)^2$.
- Equilibrium nonradial magnetic field, $\mathbf{B}_0 = B_0 \left(\hat{\boldsymbol{\phi}} \cos \chi + \hat{\boldsymbol{z}} \sin \chi \right)$.
- ▶ Isotherms along magnetic field lines, $T_0 \equiv T_0(R)$.
- Equilibrium profiles of temperature and pressure,

$$\alpha_T = -H \frac{\partial \ln T_0}{\partial R}, \quad \alpha_P = -H \frac{\partial \ln p_0}{\partial R}, \quad H = \sqrt{\frac{p_0/\rho_0}{GM/R^3}}.$$

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Stability Analysis

- Examine axisymmetric nonradial modes, $\delta a \propto \exp(ik_Z z + \Gamma t)$.
- Equilibrium such that system is Schwarzchild stable (specific entropy increases outwards).
- Collisional (fluid) limit: magnetoviscous-thermal instability (MVTI).
- Collisionless limit: collisionless MTI.
- ► Estimated modal radial angular momentum fluxes $\langle T_{R\phi} \rangle$ and heat fluxes $\langle F_{ER} \rangle$ of instabilities:

$$\begin{split} \langle T_{R\phi} \rangle &= \mathsf{Re} \left(\rho_0 \delta u_R \delta u_{\phi}^* - \frac{\delta B_R \delta B_{\phi}^*}{4\pi} + \delta \sigma_{\mathbf{b}\mathbf{b}} \delta b_R^* \cos \chi \right), \\ \langle F_{ER} \rangle &= \mathsf{Re} \left(\frac{5}{2} \rho_0 \delta \theta \delta u_R^* + \delta q \delta b_R^* - \frac{1}{3} \delta \sigma_{\mathbf{b}\mathbf{b}} \delta b_R^* \right). \end{split}$$

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MHD Equations

Constituent MHD equations of continuity, force balance, thermal energy, and magnetic induction in a dilute plasma:

$$\begin{split} \left(\frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}\right) \rho + \nabla \cdot (\rho \mathbf{u}) &= 0, \\ \rho \left(\frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}\right) \mathbf{u} + 2\Omega \hat{\mathbf{z}} \times \rho \mathbf{u} + \Omega' R \rho u_R \hat{\phi} &= -\nabla \left(\frac{B^2}{8\pi}\right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} + \nabla \cdot \left(\frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}\right) \mathbf{u} + 2\Omega \hat{\mathbf{z}} \times \rho \mathbf{u} + \Omega' R \rho u_R \hat{\phi} = -\nabla \left(\frac{B^2}{8\pi}\right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} + \nabla \cdot \left(\frac{\partial}{2\pi} + \Omega \frac{\partial}{\partial \phi}\right) \mathbf{u} + \nabla \cdot \mathbf{u} = \nabla \cdot (\mathbf{q} \mathbf{b}) + \nabla \cdot \mathbf{u} = \nabla \cdot (\mathbf{q} \mathbf{b}) + \nabla \mathbf{u} = \nabla \cdot (\mathbf{c} \mathbf{b} + \nabla \mathbf{$$

Forms of σ_{bb} and q whether *fluid* or *kinetic* analysis.

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Analysis

▶ viscous stress and parallel heat flux are given by the following, where η_{κ} and η_{ν} are (electron) thermal and (ion) viscous diffusivities whose values are given by Braginskii (1965),

$$q = \frac{5}{2} \eta_{\kappa} \mathbf{b} \cdot (k_B T_e / m_e),$$

$$\sigma_{\mathbf{b}\mathbf{b}} = 3\rho \eta_{\nu} \left(\mathbf{b} \cdot \nabla \mathbf{u} - \frac{1}{3} \nabla \cdot \mathbf{u} + \Omega' R b_R \hat{\boldsymbol{\phi}} \right).$$

- ► Fluid treatments applicable to regions where $T_i \simeq T_e$, therefore $Pr^{-1} = \eta_\kappa / \eta_\nu = 101$.
- Analysis done in the incompressible limit largely valid where $v_A \lesssim \theta_0^{1/2}$.

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Dispersion Relation



Plot of the growth rate as a function of wavenumber for various α_T for a Keplerian rotation profile and $\eta_{\nu}\Omega/v_{A}^{2} = 10^{2}$.

Plot of the dispersion relation for various α_T for a Keplerian rotation profile and a small viscous diffusion coefficient $\eta_{\nu}\Omega/v_A^2 = 1$.

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| MVTI | | | | | | |

Fluxes for MVI-Like Modes



Normalized radial flux of thermal energy for a Keplerian rotational profile and $\eta_{\nu}\Omega/v_{A}^{2} = 10^{2}$.

Normalized angular momentum flux for a Keplerian rotational profile and $\eta_{\nu}\Omega/v_{A}^{2} = 10^{2}$.

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Fluxes for MRI-Like Modes



Normalized radial flux of thermal energy for a Keplerian rotational profile and $\eta_{\nu}\Omega/v_{A}^{2} = 1$.

Normalized angular momentum flux for a Keplerian rotational profile and $\eta_{\nu}\Omega/v_{\pmb{A}}^{\pmb{2}}=1.$

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| Collisionless M | Т | | | | | |

Basics of Collisionless MHD in Rotating Frame

One works with the Boltzmann equation with, at most, mild collisionality,

$$\frac{\partial f}{\partial t} + \mathbf{U} \cdot \nabla f + \left(\mathbf{g} + \frac{Z_s e}{m_s} \left[\mathbf{E} + \frac{1}{c} \mathbf{U} \times \mathbf{B} \right] \right) \cdot \frac{\partial f}{\partial \mathbf{U}} = C[f]$$

Corotating, magnetic-field centered velocity transformation, $\mathbf{U} \rightarrow \left(\mathbf{v}_{||}, \mu, \psi\right)$:

$$\begin{split} \mathbf{v}_{\parallel} &= \mathbf{U} \cdot \mathbf{b} - R\Omega b_{\phi} \\ \mu &= \frac{\left(\mathbf{U} - R\Omega \hat{\phi} + R\Omega b_{\phi} \mathbf{b} - \mathbf{b} \left(\mathbf{U} \cdot \mathbf{b}\right) - \mathbf{u}_{\perp}\right)^{2}}{2B} \\ \tan \psi &= \frac{\hat{\mathbf{y}}_{\perp} \cdot \left(\mathbf{U} - R\Omega \hat{\phi} - \mathbf{u}_{\perp}\right)}{\hat{\mathbf{x}}_{\perp} \cdot \left(\mathbf{U} - R\Omega \hat{\phi} - \mathbf{u}_{\perp}\right)}. \end{split}$$

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| Collisionless MTI | | | | | | | | |

After some algebra, and given our equilibrium, the lowest-order Boltzmann equation is the drift-kinetic equation (Kulsrud, 1983, 2005):

$$\begin{split} \left(\frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}\right) f_{s} + \left(\mathbf{v}_{\parallel} \mathbf{b} + \mathbf{u}_{\perp}\right) \cdot \nabla f_{s} + \left(\frac{Z_{s} e E_{\parallel}}{m_{s}} + \frac{m_{p}}{\rho_{0} m_{s}} \mathbf{b} \cdot \nabla \rho_{s0}\right) \frac{\partial f_{s}}{\partial v_{\parallel}} + \\ \left(-\mathbf{b} \cdot \left(\left[\frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}\right] \mathbf{u}_{\perp} + \left[\mathbf{u}_{\perp} + v_{\parallel} \mathbf{b}\right] \cdot \nabla \mathbf{u}_{\perp}\right) + \frac{1}{2} v_{\perp}^{2} \nabla \cdot \mathbf{b}\right) \frac{\partial f_{s}}{\partial v_{\parallel}} + \\ 2\Omega \hat{z} \cdot \left(\mathbf{b} \times \mathbf{u}\right) - \Omega' R b_{\phi} \hat{\mathbf{R}} \cdot \left(\mathbf{u}_{\perp} + v_{\parallel} \mathbf{b}\right) \frac{\partial f_{s}}{\partial v_{\parallel}} = C \left[f_{s}\right]. \end{split}$$

Parallel and perpendicular pressures to the magnetic field, p_{\parallel} and p_{\perp} ,

$$p = \frac{2}{3}p_{\perp} + \frac{1}{3}p_{\parallel}, \qquad \sigma_{\mathbf{b}\mathbf{b}} = p_{\parallel} - p_{\perp},$$
$$p_{s\parallel} = 2\pi m_s \int f_s v_{\parallel}^2 \, dv_{\parallel} \, v_{\perp} \, dv_{\perp}, \quad p_{s\perp} = 2\pi m_s \int f_s \left(\frac{1}{2}v_{\perp}^2\right) \, dv_{\parallel} \, v_{\perp} \, dv_{\perp}.$$

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| Collisionless MTI | | | | | | | |

Dispersion Relation

At high β , collisionless MTI appears qualitatively similar to MVTI. Below are plots of the dispersion relation for $\beta = 10^2$, $\chi = \pi/4$, $\alpha_P = 5$, and $0 \le \alpha_T \le 2$.







The imaginary part of the growth rate for the collisionless MTI with various equilibrium temperature and pressure profiles.

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| Collisionless MTI | | | | | | | |

Dispersion Relation

Collisionless damping of long wavelength modes $k_{\parallel} < \Omega/v_i$ (see, e.g., Quataert et al. (2002); Sharma et al. (2003)). As $\beta \rightarrow 1$, anisotropic pressure becomes dynamically unimportant.



The real component of the growth rate for various β , $\chi = \pi/4$, $\alpha_{P} = 5$, and $\alpha_{T} = 2$. For large β we reproduce MRI-like modes.



The imaginary component of the growth rate for various β , $\chi = \pi/4$, $\alpha_P = 5$, and $\alpha_T = 2$. For large β the imaginary component reaches a maximum where the growth rate reaches a maximum. As $\beta \rightarrow 1$, the imaginary component of the growth rate increases in magnitude.

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Modal Fluxes

Modal fluxes $\langle F_{ER} \rangle$ and $\langle T_{R\phi} \rangle$ for a Keplerian-like rotation profile, $\beta = 10^2$, and $\chi = \pi/4$, $\alpha_P = 5$, and $0 \le \alpha_T \le 2$.



Normalized angular momentum flux for collisionless Normalized heat flux for collisionless MTI. MTI.

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Mildly Collisional MTI

In the limit that mean free path is larger than wavelength of fastest growing modes, i.e., $\nu \gtrsim \Omega \beta^{1/2}$, we reproduce the MVTI (see, e.g., Sharma et al. (2003)).



Real (left) and imaginary (right) parts of the growth rate of the MVTI. We reproduce the MVTI dispersion relation for $\nu_i \geq \Omega \beta^{1/2}$.

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| Collisionless MTI | | | | | | | |

Finite Electron Temperature Effects

Not significant different in growth rates if we include electron thermal dynamics in the dispersion relation.



The real (left) and imaginary (right) parts of growth rate of the collisionless MTI. The imaginary part of the growth rates where $T_{e0}/T_{i0} = 10^{-1}$ nearly coincide where $T_{e0}/T_{i0} = 0$.

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| Collisionless MTI | | | | | | | |

Off-Plane Collisionless MHD Effects

We take $\beta = 10^2$. At left is the real, and at right is the imaginary, normalized growth rate as a function of height z/H for $k_Z v_{A0}/\Omega = 10^{-1}$. We compare Γ to a simplified growth rate, Γ_s , where effects due to equilibrium acceleration are ignored.



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Improved Landau Fluid Closures Necessary Standard forms of heat fluxes in a collisionless MHD plasma (Snyder et al., 1997):

$$\frac{\delta q_{\parallel}}{\rho_{i0} v_{i}} = 2i\sqrt{\frac{2}{\pi}} \left(\frac{\delta \rho}{\rho_{0}} - \frac{\delta \rho_{\parallel}}{\rho_{i0}}\right) - 2\left(\frac{1}{k_{\parallel}}\frac{\partial \ln T_{0}}{\partial R}\right)\sqrt{\frac{2}{\pi}}\frac{\delta B_{R}}{B_{0}}$$
$$\frac{\delta q_{\perp}}{\rho_{i0} v_{i}} = i\sqrt{\frac{2}{\pi}} \left(\frac{\delta \rho}{\rho_{0}} - \frac{\delta \rho_{\perp}}{\rho_{i0}}\right) - \left(\frac{1}{k_{\parallel}}\frac{\partial \ln T_{0}}{\partial R}\right)\sqrt{\frac{2}{\pi}}\frac{\delta B_{R}}{B_{0}}$$



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Isotropizing Gyrokinetic Instabilities

- Treatment of firehose and mirror instabilities considered as a "hard wall" on pressure anisotropy by Sharma et al. (2006); Sharma (2006); Sharma et al. (2007) for collisionless MRI.
- Treatments of collisionless turbulence (Schekochihin & Cowley, 2005; Howes et al., 2007a,b) may need to address this in order to accurately describe dynamics of collisionless MHD plasmas.



Density histogram of measurements of the solar wind by the WIND telescope. Fits of histogram to threshold instability to cyclotron and mirror instabilities, as determined by particle simulations of Gary et al. (1996, 1997). This figure is taken from Hellinger et al. (2006).

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Necessity of Global Simulations



Secular increase of total energy within a radial slice for simulation of the MRI. Figure taken from Gardiner & Stone (2005). Global simulations are **necessary** for modeling radiatively inefficient flows; codes that explicitly conserve all energy, such as Athena (Gardiner & Stone, 2005), are necessary to model the nonlinear collisionless MTI or MVTI.

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Main Points

- Both collisionless MTI and MVTI can provide significant outward heat and angular momentum fluxes to drive accretion in radiatively inefficient flows.
- Decreasing β (increasing relative strength of magnetic field) results in decreased anisotropic fluxes of heat and momentum in linear collisionless MTI.
- Simplified analysis (neglect of electron temperature) is valid in case where T_i ≥ T_e; further work needed to characterize the effects of finite equilibrium acceleration in collisionless off-plane MTI.

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