The Magnetoviscous Instability With General Viscosity

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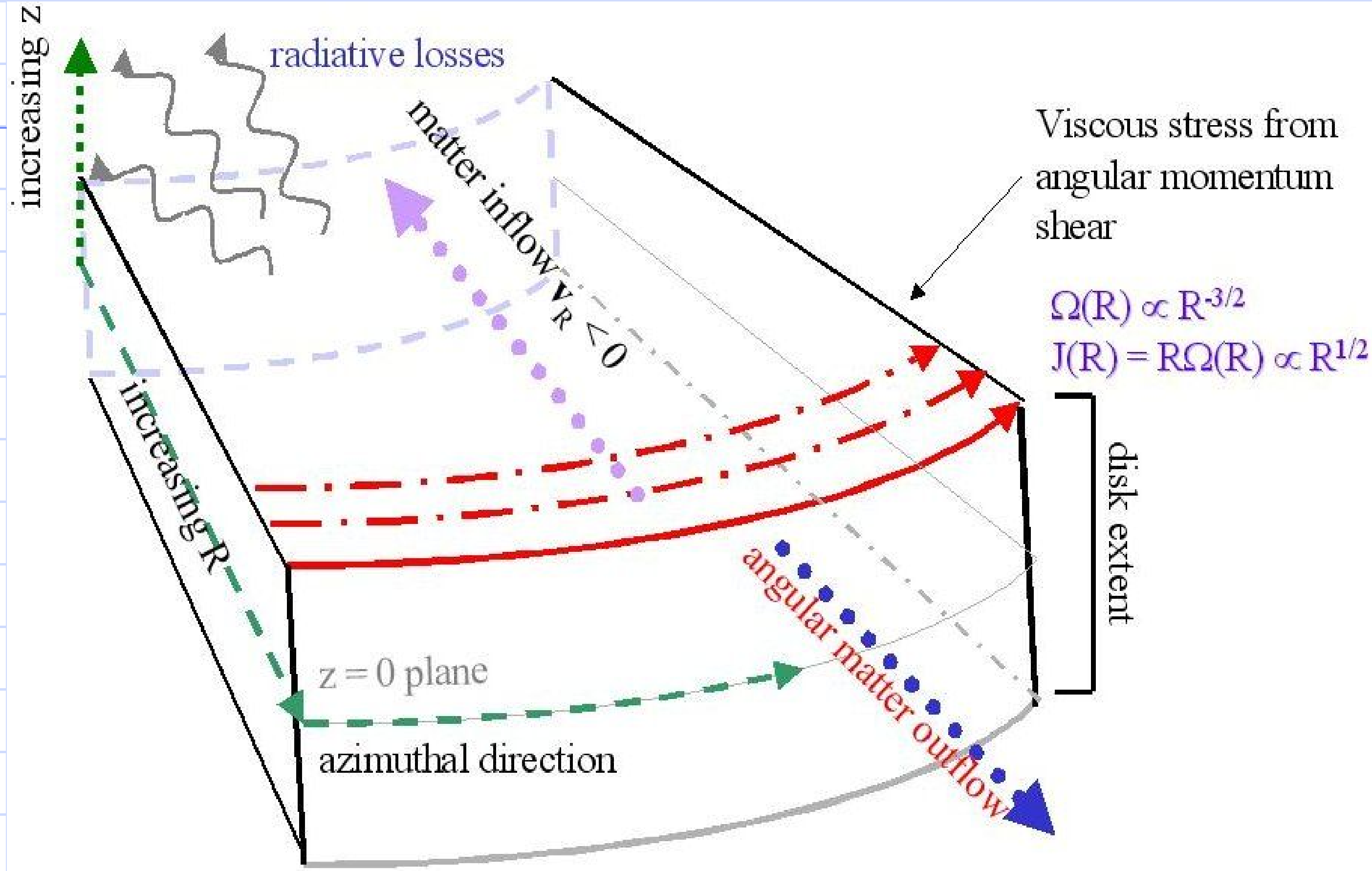
Phenomenological Model of Rotating Astrophysical Disks

- transfer of angular momentum (spin) **outwards**
- transfer of matter **inwards**
- Process is mediated by a "**phenomenological**" viscosity, not the perfectly-understood viscosity arising from particle-particle collisions



α Viscosity Paradigm

Shakura and Sunyaev [1] believed diffusion was enhanced by ***hydrodynamic turbulence*** – the size of the cells is **H** (disk thickness); the sound crossing speed is $\mathbf{c_s}$ (sound speed)

$$\eta_\nu = \alpha c_s H \gg c_s^2/\nu_{ii}$$

α	dimensionless parameter
ν_{ii}	ion-ion collision rate
c_s^2/ν_{ii}	order-of-magnitude estimate of the viscosity, resulting in accretion timescales of order $10^{10} - 10^{12}$ years

Example: Diffusion Equation For Accretion

$$
\begin{aligned}
v_R(R,t) &= \frac{1}{R\Sigma d\left(R^2\Omega\right)/dR}\frac{\partial}{\partial R}\left(\eta_\nu \Sigma R^3 \frac{d\Omega}{dR}\right) \\
\frac{\partial \Sigma}{\partial t} &= -R^{-1}\frac{\partial}{\partial R}\left(R\Sigma v_R\right) \equiv -\frac{1}{R}\frac{\partial}{\partial R}\left[\frac{1}{d\left(R^2\Omega\right)/dR}\frac{\partial}{\partial R}\left(\eta_\nu \Sigma(R) R^3 \frac{d\Omega}{dR}\right)\right] \\
\Sigma(R,t) &= \text{Surface mass density} \\
v_R &= \text{inflow accretion velocity}
\end{aligned}
$$

- Above diffusion equation is applied to flows within thin accretion disks, but paradigm is universal in disk accretion models.
- η_ν is a phenomenological α viscosity.

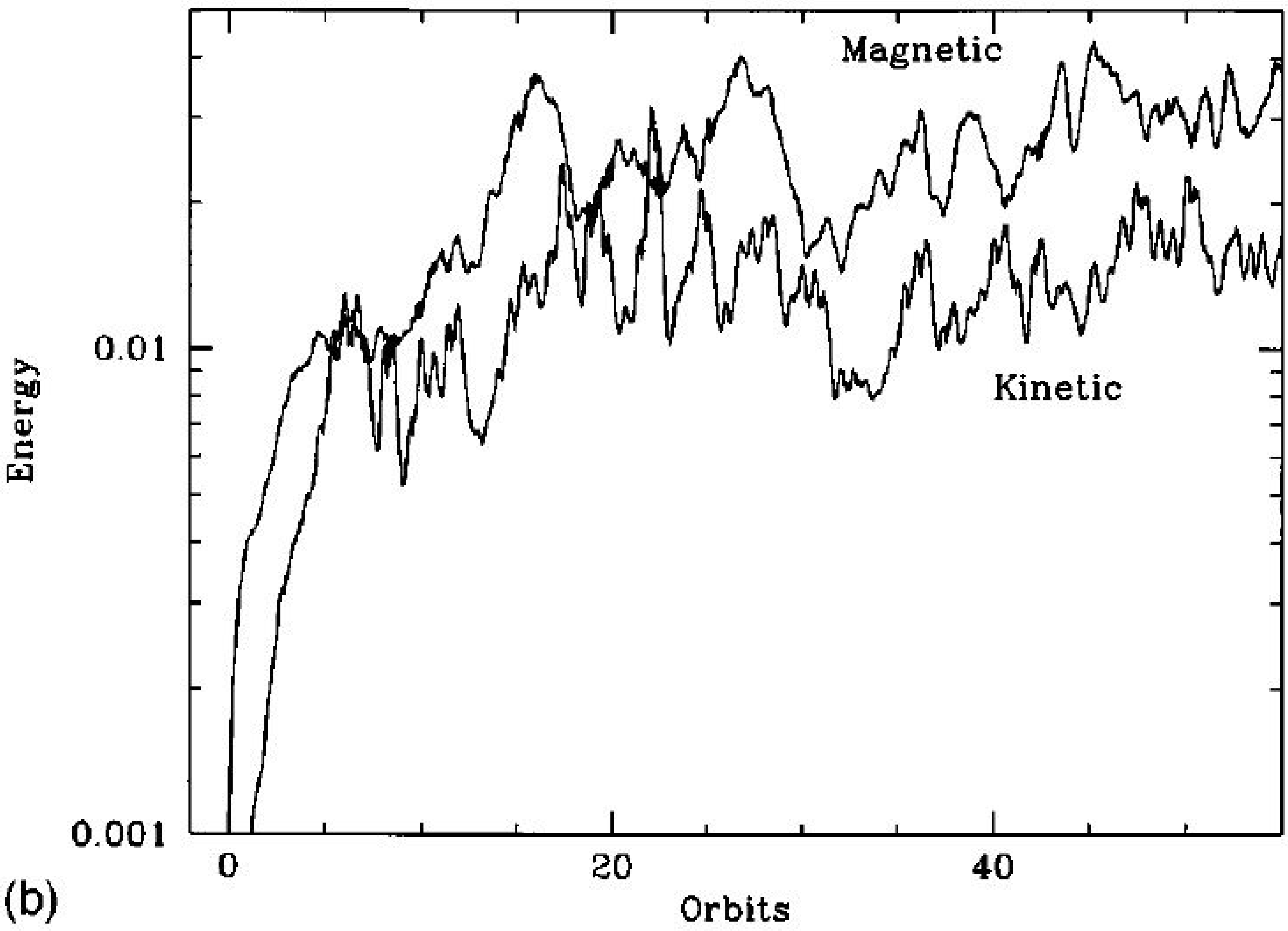
The Magnetorotational Instability

- First discovered by Velikhov [2] and Chandrasekhar [3], and used as an explanation for rigid-body (constant Ω) rotation in stars.
- Magnetized disks in which decreasing outwards **angular velocity** Ω rather than **angular momentum** ΩR^2 (stability criterion for hydrodynamic disks) destabilize the disk.
- Instability grows at the rate of Ω at wavelengths much **smaller** than the disk height (“turbulence” within the disk arising from magnetic fields).

Astrophysical Application

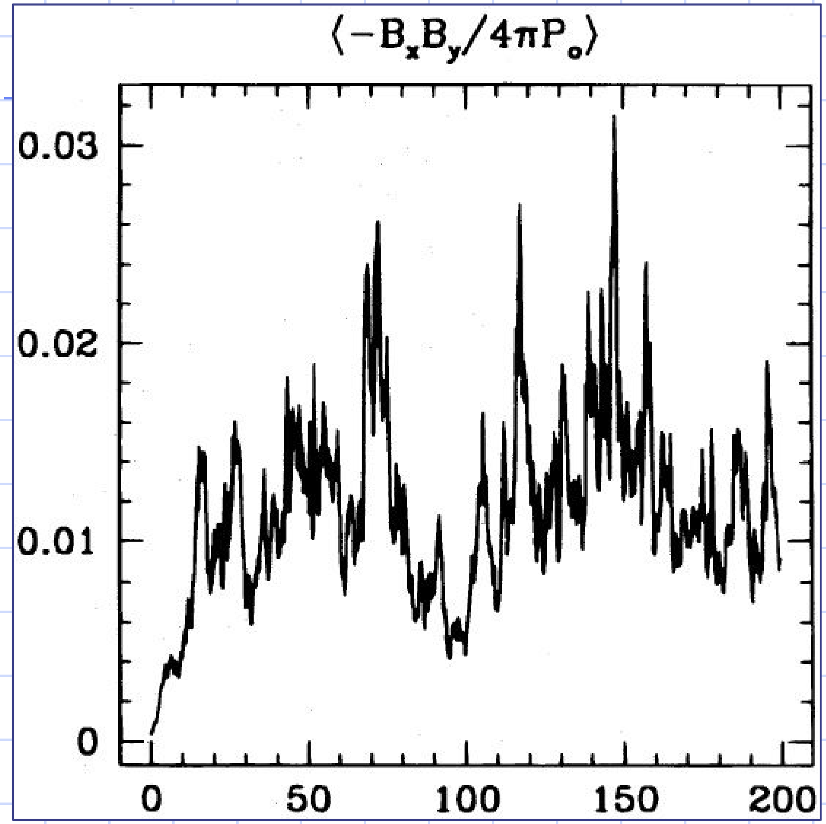
- Balbus and Hawley [4] showed that the MRI could be applied under much more general and universal conditions (namely that Ω decreases outward radially) and is a global instability (important wherever in the disk that the above condition is met).
- First to apply the use of the MRI in explaining magnetized turbulence, hence enhanced viscosity, within accretion disks.
- From 2D and 3D simulations, showed that magnetic fields from even a weak level saturate at pressures comparable to the gas pressure.
- Numerically simulated $\alpha \sim 1$ (or not much smaller).

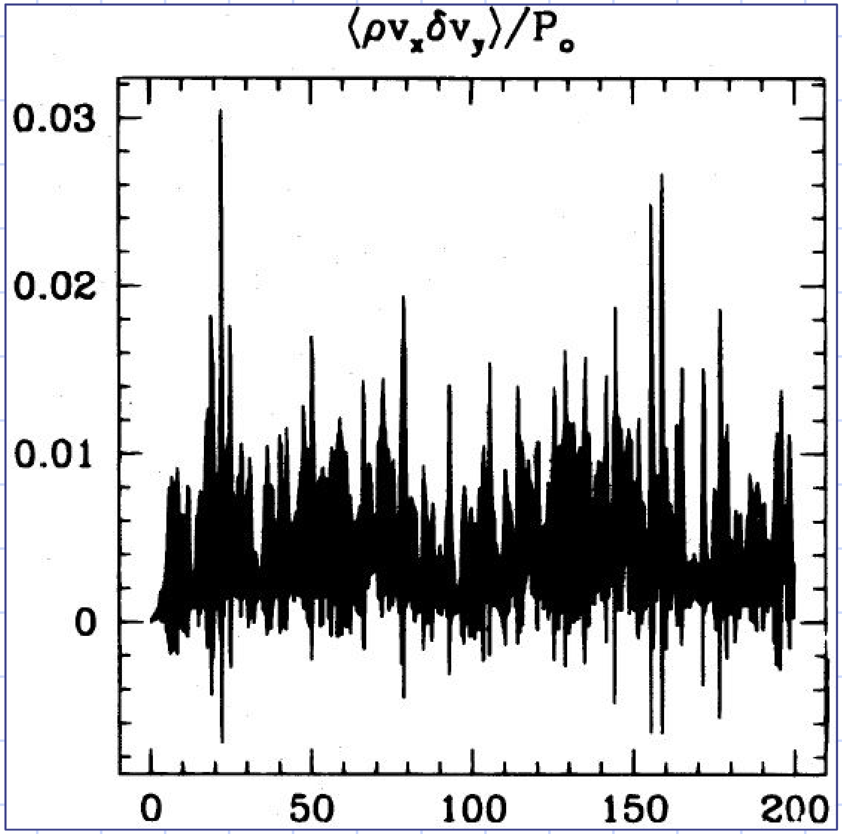
Magnetic Field Saturation [5]



(b)

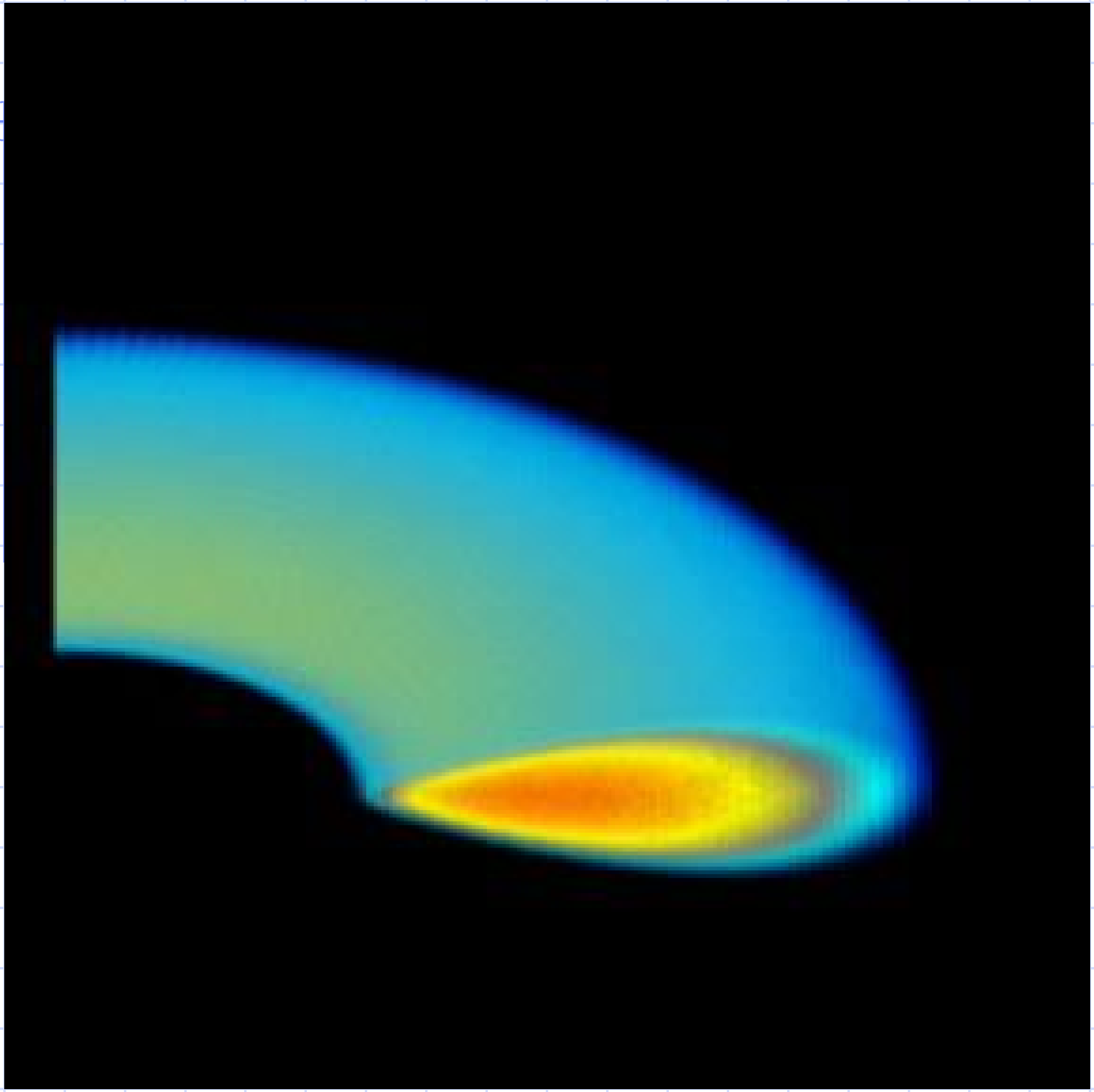
Significant Viscous Stresses [6]





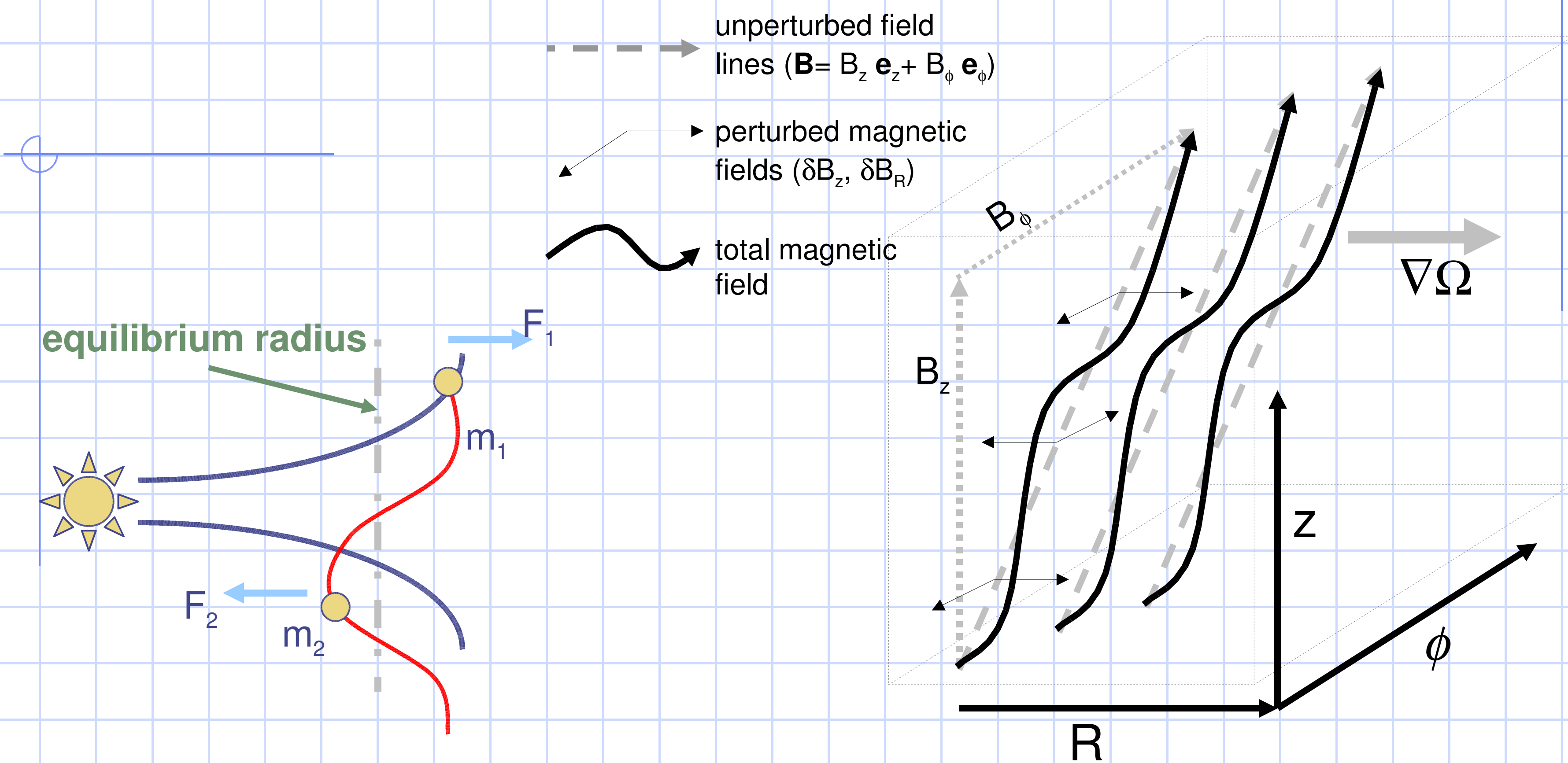
$\alpha \sim 10^{-2}$ in above simulations

Nonlinear Simulation of MRI



Taken from http://www.astro.virginia.edu/VITA/papers/plunge

Schematic Model of MRI



- Points on a magnetic field line are forced to corotate (same Ω).
- The points further out from the equilibrium tend to accelerate outward, while points inside accelerate inwards.
- QUENCHED at small enough wavelengths due to the "springiness" of magnetic tension.

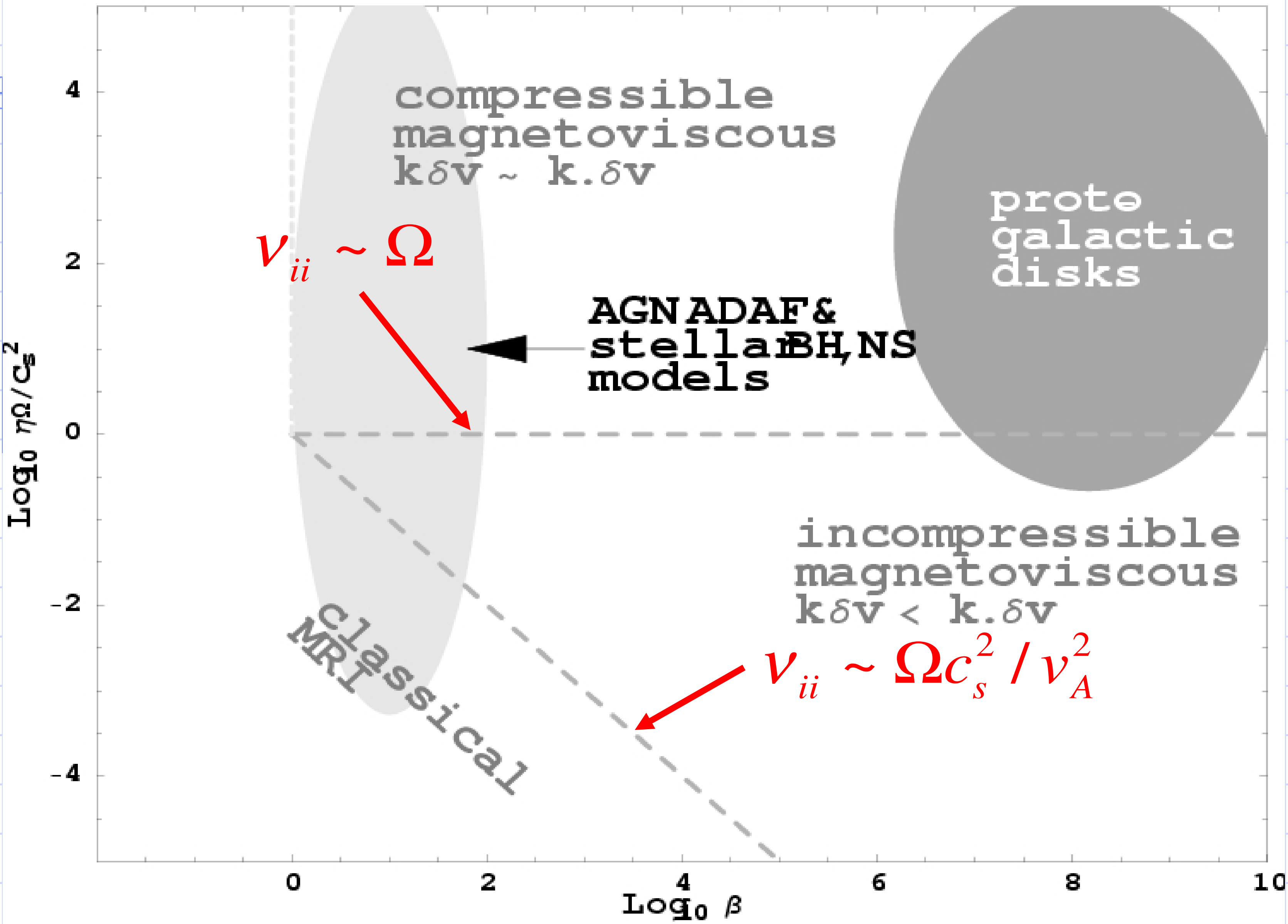
Magnetoviscous Instability (MVI)

- Weak magnetic fields, so no magnetic forces as in MRI.
- Strong enough magnetic field ($\nu_{ii} < \Omega_{ci}$) to anisotropize the viscosity along the magnetic field line [7].
- Saturation of mode at wavelengths $\lambda \sim (\eta_\nu/\Omega)^{1/2}$, much longer than MRI saturation wavelength $\lambda > v_A/\Omega$.
- Physical differences between MRI and MVI.
  - MRI: fluid tether through **magnetic** force.
  - MVI: fluid tether through **anisotropic viscous** force, which itself lies along magnetic field lines.

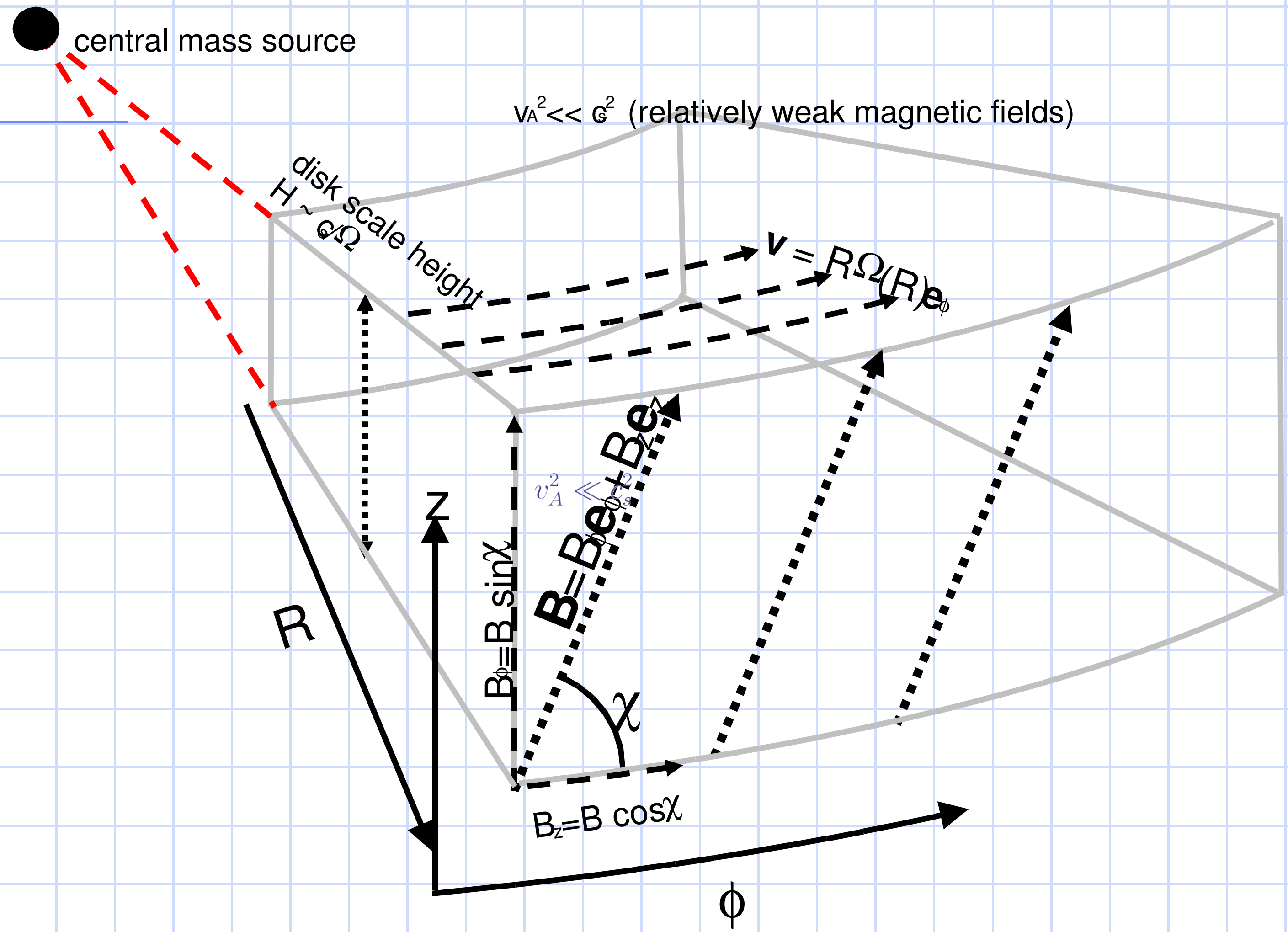
Justification for Study of the MVI

- Certain classes of rotating astrophysical objects are unstable to these modes – those that are characterized by very dilute plasmas and relatively weak magnetic fields.
  - protogalactic disks – amplification of weak magnetic fields.
  - RIAFs – very hot (ion temperatures ~ 10^{12} K), dilute, optically thin and nonradiative plasma around compact objects.
- May allow for growth of magnetic fields in ISM to thermal strengths, as well as provide a mechanism for turbulent α viscosity in RIAFs.

Astrophysical Objects Unstable to the MVI



Equilibrium Disk



- **B**$=B_\phi$**e**$_\phi+B_z$**e**$_z$ (no steady-state magnetic shearing)
- Purely rotational eq. velocity: **v**$= R\Omega$ **e**$_\phi$
- the magnetic field is highly **subthermal**: $v_A^2 << c_s^2$

Unstable Mode Analysis

- axisymmetric instabilities, $\delta a \propto \exp\left(ik_z z + ik_R R + \Gamma t\right)$

  where δa is perturbed quantity, Γ is growth rate, and k_R and k_z are radial and vertical wavenumbers.
- **Boussinesq approximation – incompressible instabilities** $\nabla \cdot \delta v = 0$
- WKB (wave) approximation, examining wavelengths < H (k H > 1).
- Useful normalizations:

$$\hat{\mathbf{k}} = \mathbf{k}H = \mathbf{k}c_s/\Omega$$

$$\hat{\Gamma} = \Gamma/\Omega$$

$$\hat{\eta}_\nu = \eta_\nu/\left(c_s H\right) = \eta_\nu \Omega/c_s^2$$

Dispersion Relation

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} = 0$$

$$A_{11} = \hat{\Gamma}^2 \left(1 + \frac{k_R^2}{k_z^2}\right) + 3\hat{\eta}_\nu \hat{k}_R^2 \hat{\Gamma} \sin^4\chi + 2\frac{d\ln\Omega}{d\ln R} + \beta^{-1}\left(\hat{k}_R^2 + \hat{k}_z^2\right)\sin^2\chi$$

$$A_{12} = -\left(2 + 3\hat{\eta}_\nu \hat{k}_R \hat{k}_z \sin^3\chi\cos\chi\right)\hat{\Gamma}$$

$$A_{13} = \frac{\hat{k}_R}{\hat{k}_z}\hat{\Gamma}^2 \sin\chi - 2i\hat{\Gamma}\left(\hat{\eta}_\nu \hat{k}_R \hat{k}_z \sin^3\chi + \cos\chi\right)$$

$$A_{21} = \left(2 - 3\hat{\eta}_\nu \hat{k}_R \hat{k}_z \sin^3\chi\cos\chi\right)\hat{\Gamma}$$

$$A_{22} = \hat{\Gamma}^2 + 3\hat{\eta}_\nu \hat{k}_z^2 \sin^2\chi + \beta^{-1}\hat{k}_z^2 \sin^2\chi$$

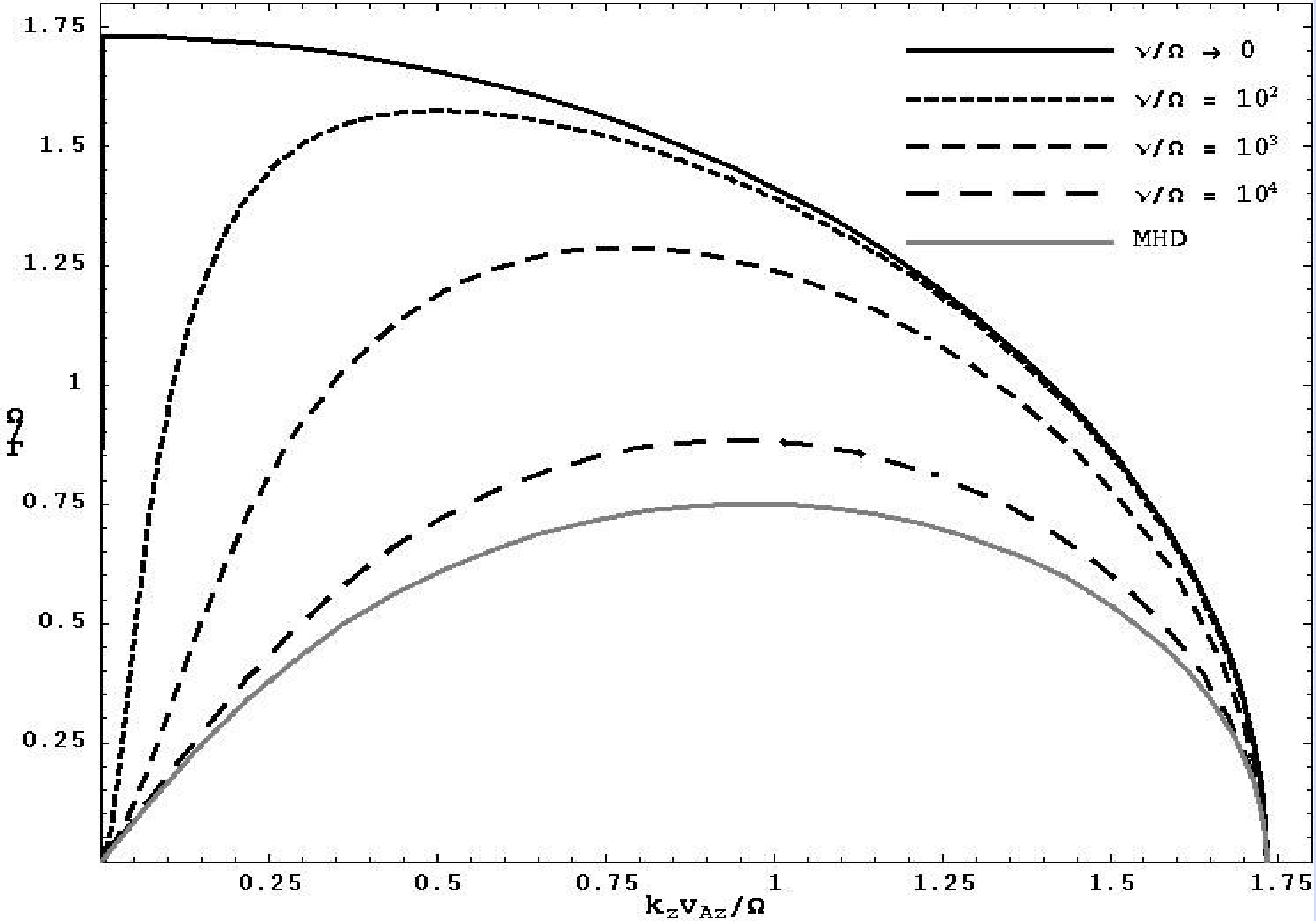
$$A_{23} = i\hat{\Gamma}\cos\chi\left(\hat{\Gamma} + 2\hat{\eta}_\nu \hat{k}_z^2 \sin^2\chi\right)$$

$$A_{31} = \frac{\hat{k}_R}{\hat{k}_z \sin\chi}\hat{\Gamma}^2 + \hat{\eta}_\nu \hat{k}_R \hat{k}_z \hat{\Gamma}\left(3\sin^2\chi - 1\right)\sin\chi$$

$$A_{32} = -\hat{\eta}_\nu \hat{k}_z^2 \left(3\sin^2\chi - 1\right)\hat{\Gamma}\cos\chi + \beta^{-1}\hat{k}_z^2 \cos\chi$$

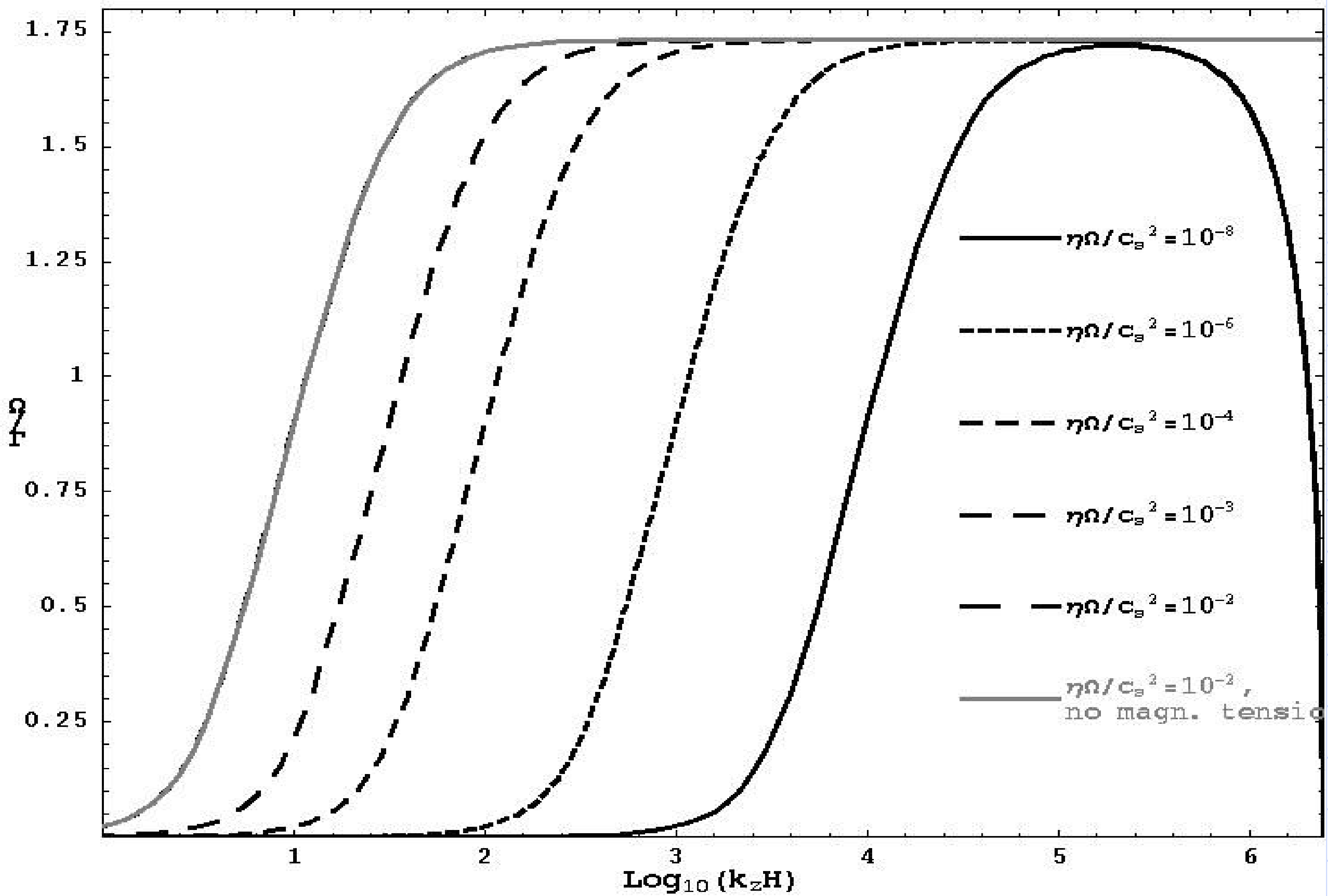
$$A_{33} = \hat{\Gamma}^2 - \frac{2}{3}i\hat{\eta}_\nu \hat{k}_z^2 \left(3\sin^2\chi - 1\right)\hat{\Gamma} + \hat{k}_z^2 \left(1 - \frac{\mathcal{P}\hat{\eta}_\nu \hat{k}_z^2 \sin^2\chi/\gamma}{\hat{\Gamma} + \mathcal{P}\hat{\eta}_\nu \hat{k}_z^2 \sin^2\chi}\right)$$

The MVI as Modification of the MRI

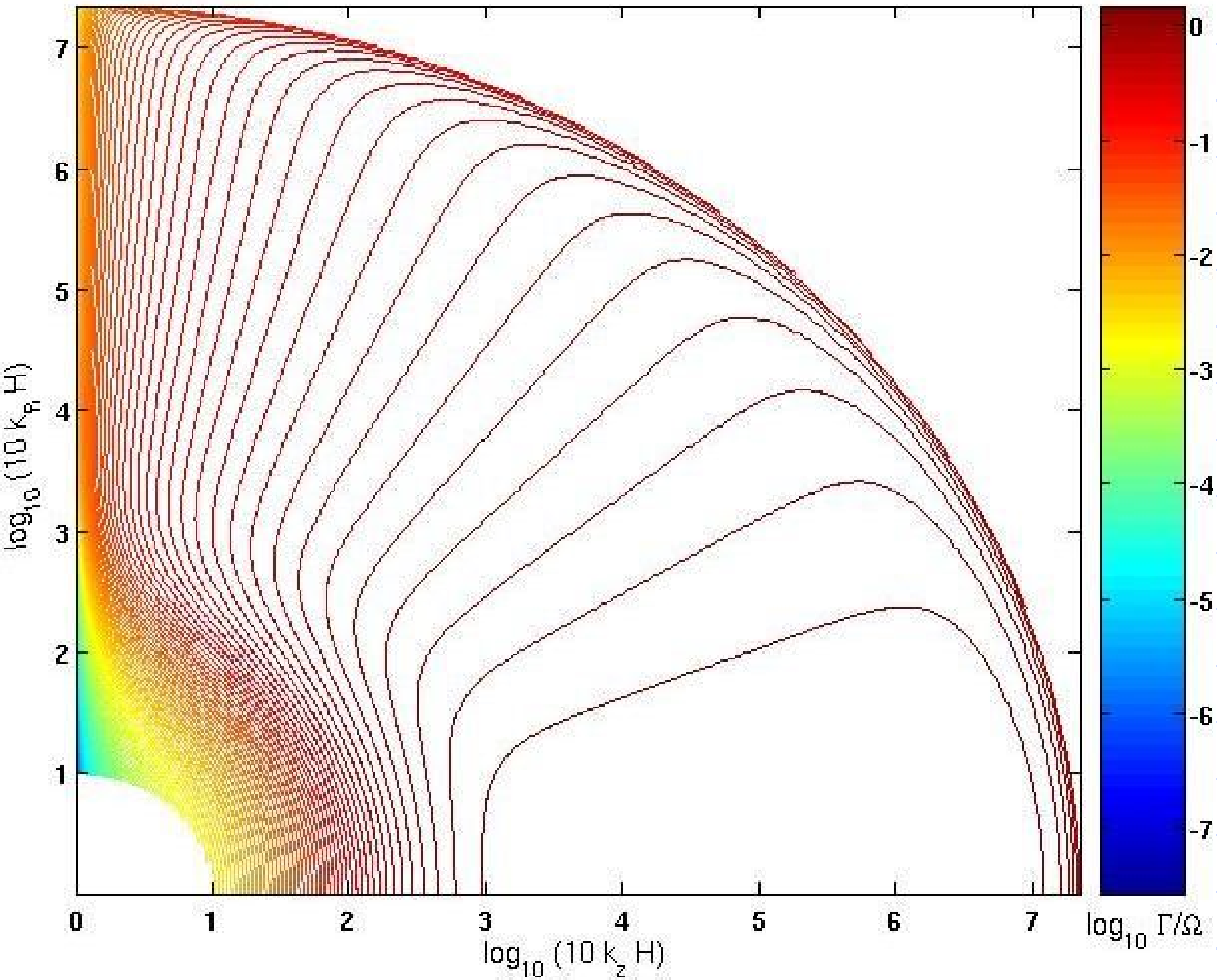


dispersion relation (growth rate as function of wavenumber) for Keplerian disk

Dispersion Relation of the MVI in More Appropriate Units

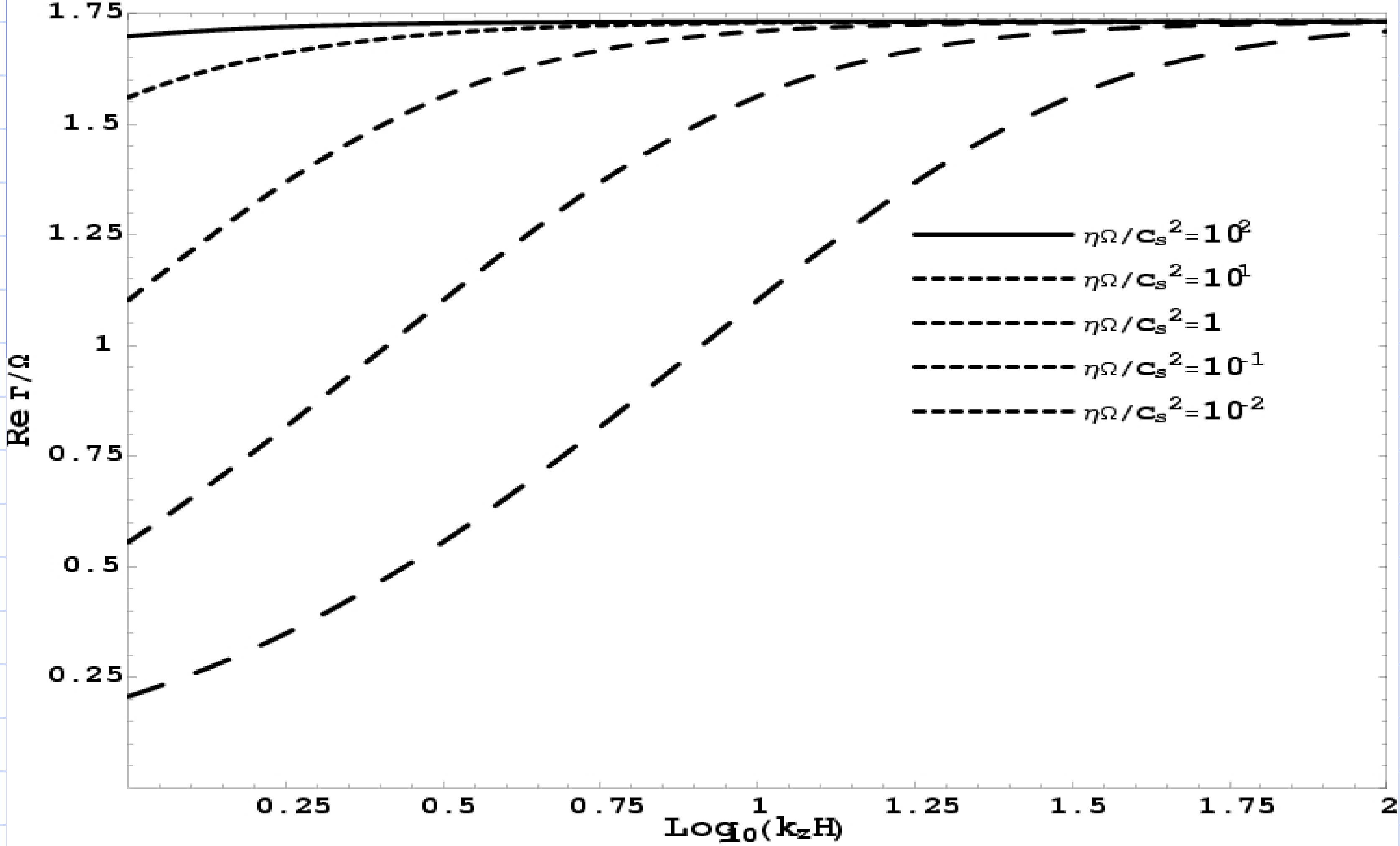


Full Axisymmetric Dispersion of MVI

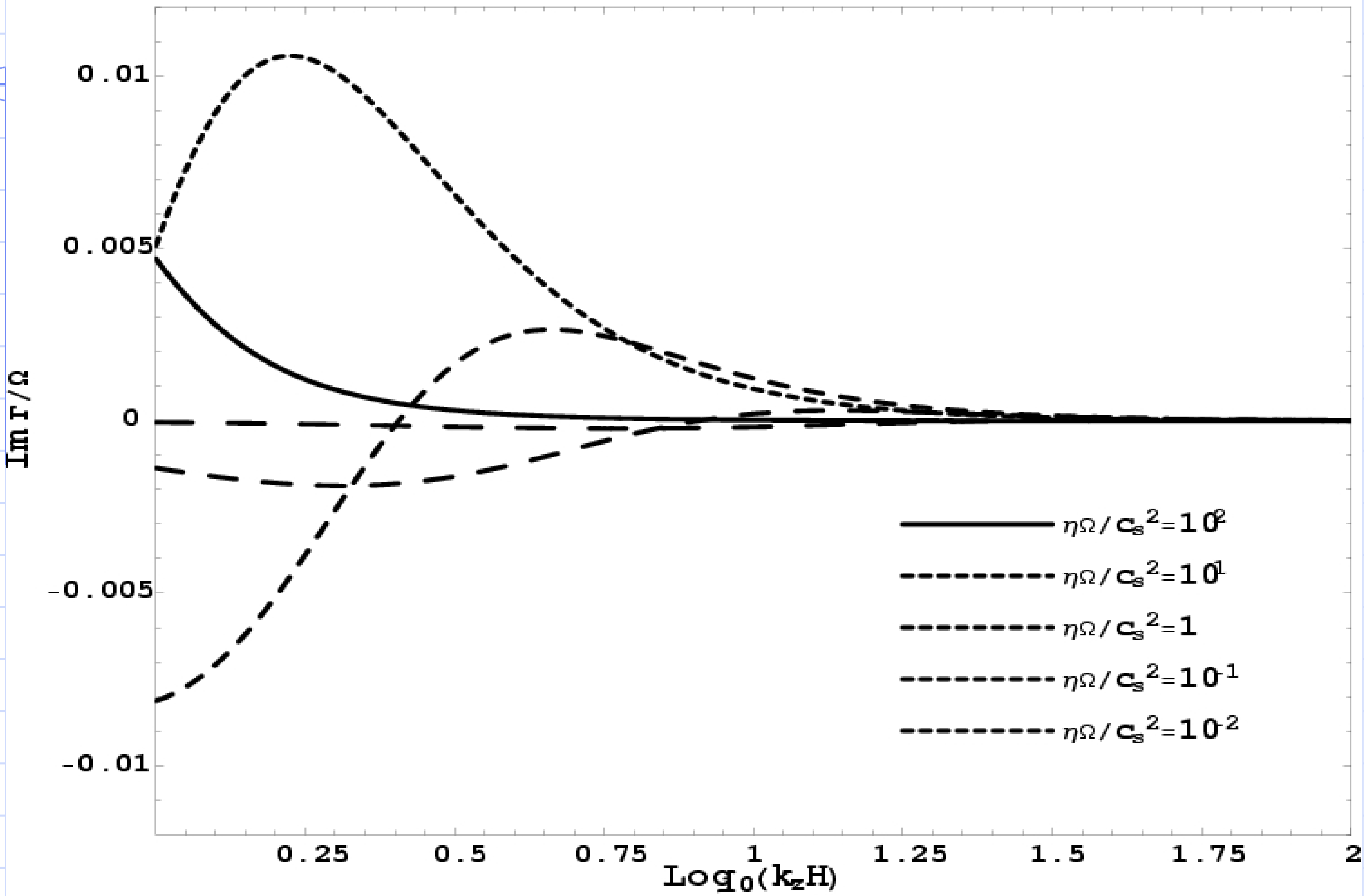


growth rate as function of wave number for $\beta=10^{12}$ plasma. Growth rate goes to zero as $k_z < k_R$.

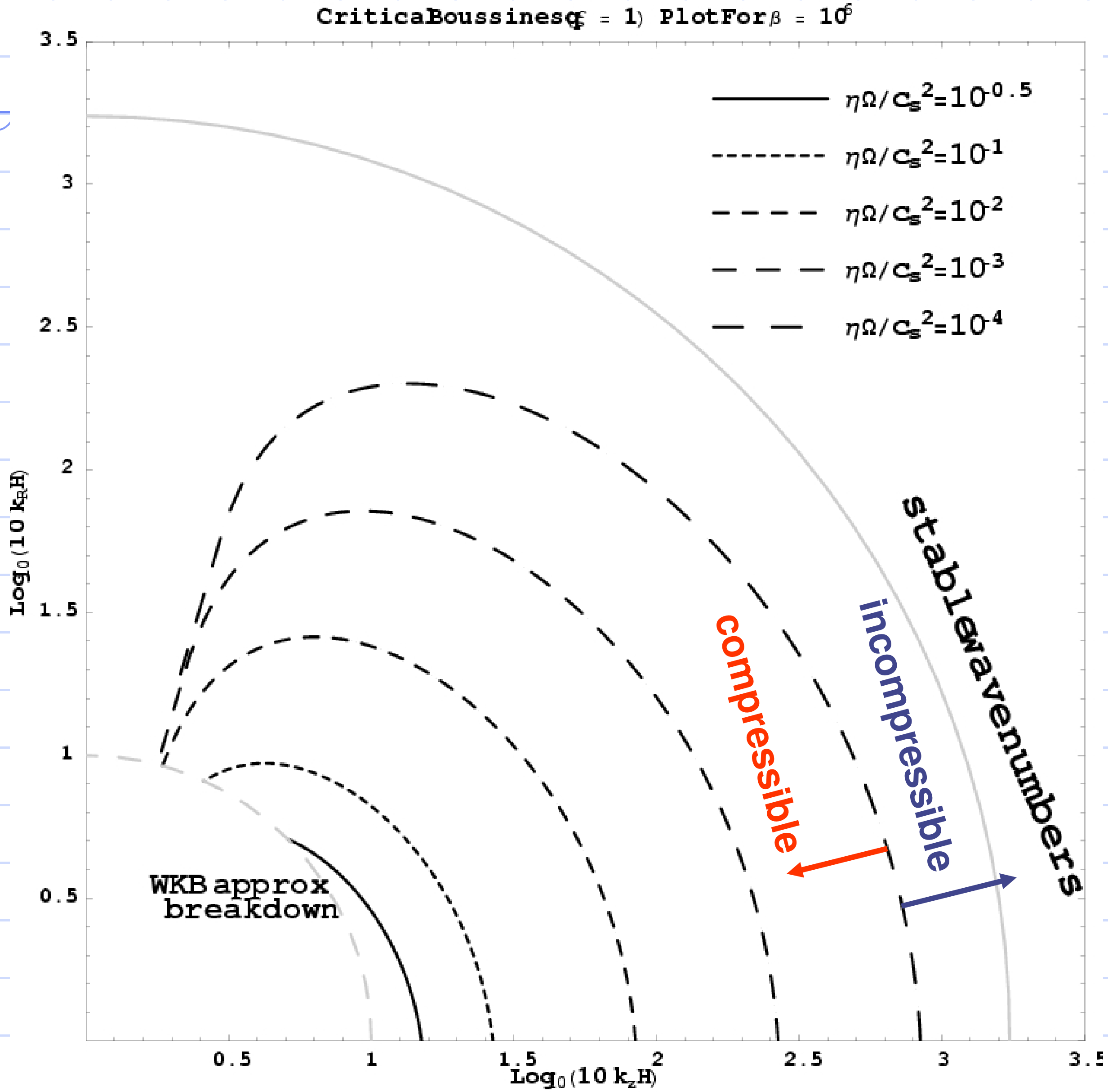
More Careful Analysis (With Finite Compressibility) Yields Similar Dispersion Relation



Oscillatory (Imaginary Part) of Growth Rate

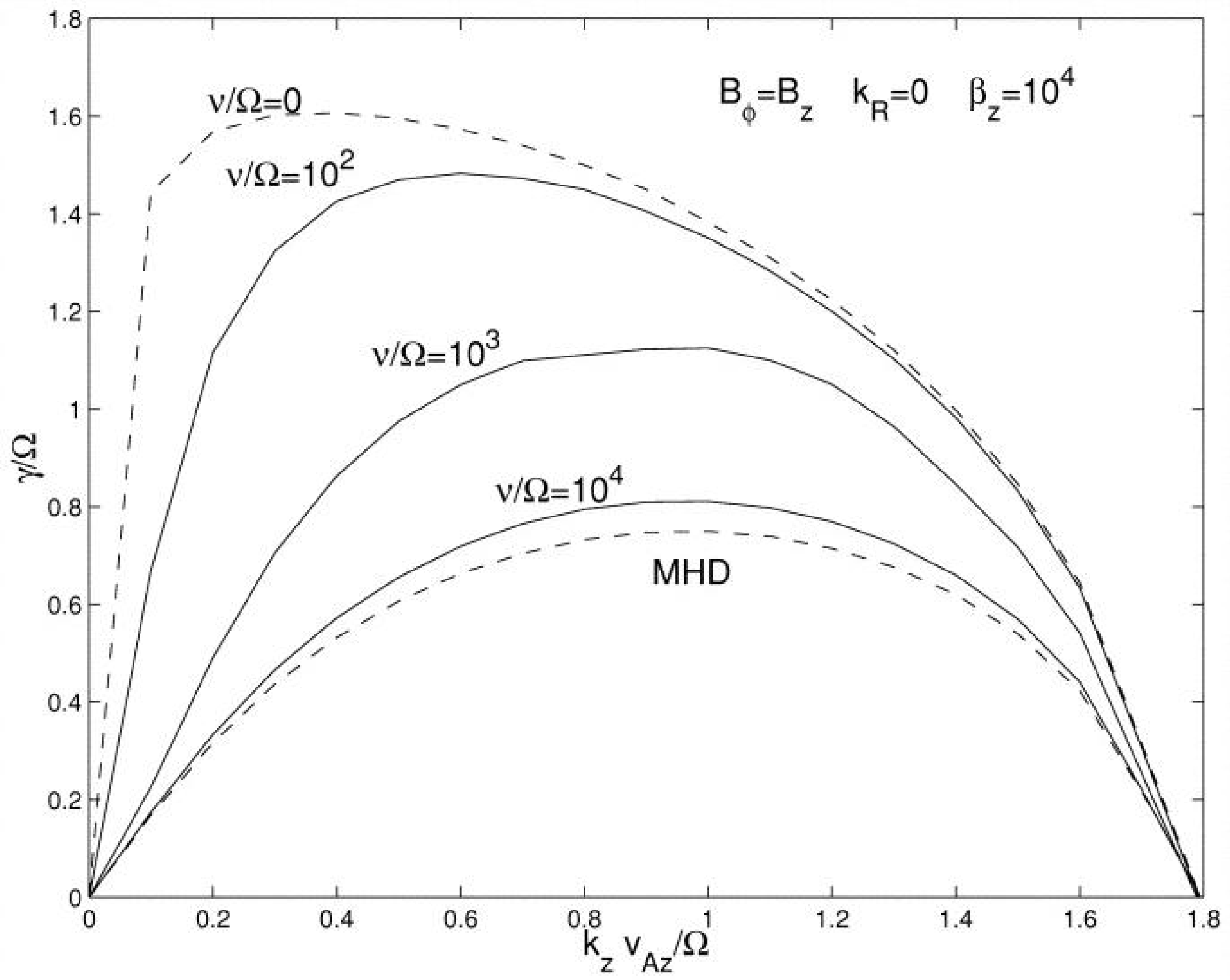


Validity of the Boussinesq Approximation



$$\xi = \frac{|k \cdot v|}{kv}$$

Kinetic Analysis Yields Results Consistent With Fluid Approach



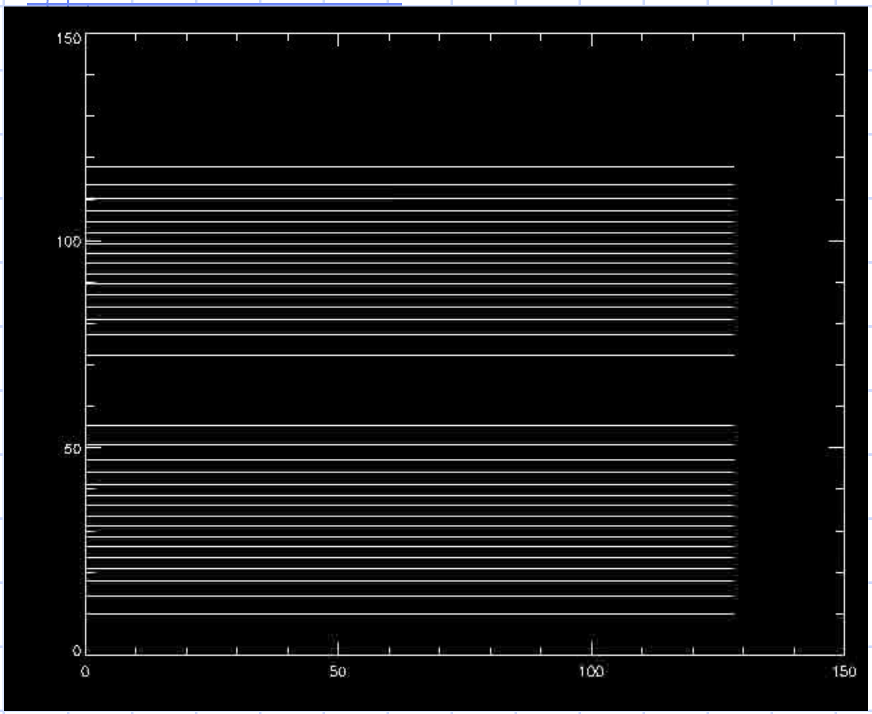
Plot taken from E. Quataert et. al. [8]. A kinetic analysis with finite collision frequency reproduces the MVI.

The MRI and MVI Are Manifestations of Changed Stability Criteria

Instability Conditions

	nonmagnetized	magnetized
angular momentum transfer	angular momentum ($\Omega^2 R$) decreases radially outwards	angular velocity (Ω) decreases radially outward
energy transfer	entropy density (heat content) decreases upward or radially outward.	temperature **T** decreases upward or radially outward – outer or upper regions cooler.

Nonlinear Thermal Instability In Magnetized Plasma



magnetic field lines

temperature

Taken from http://www.astro.princeton.edu/~iparrish

Issues With Analytic MVI

- Examine the effects of relatively large anisotropic magnetized electron thermal conductivity, as begun in [9].
- Careful kinetic study of the extremely dilute plasma ($\nu_{ii}/\Omega \rightarrow 0$) limit.
  - "knee" in the dispersion relation – result of effective wave-particle finite diffusion process?
  - Comparison to fluid approximation.
  - If "knee" exists, determine under which conditions the WKB approximation holds.
- Long-wavelength approximations in dilute plasma limit – due to saturation of MVI with WKB approximation in dilute plasma limit.

Issues With Numerical MVI

- Numerical difficulty with tracking a viscosity tied to a field parameter (magnetic field) that can vary on shortest grid lengths.
  - Preliminary numerical simulations of magnetothermal instability (MTI) demonstrate surmountability.
- Form of the power spectrum in MVI – dominated by short or long wavelengths?
  - MRI – long wavelengths, magnetic dissipative scale > viscous dissipative scale.
  - MVI – magnetic dissipative scale < viscous dissipative scale, so short(?) wavelengths as in Schekochihin et. al. [11].

References

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