Hall Astrophysics in Dusty Plasmas

- In the limit of very low plasma density and/or "fixed" ion motion (i.e. motion of ions unaffected by electromagnetic field), Hall physics may play important role, esp. when ion gyroradius is on order of or larger than scale of the problem.
- Effects of low density may imply that Hall plasma effects, such as magnetic rotation (as seen in whistler waves) and magnetic convection (through whistler drift modes) through Hall electric jet may be orders of magnitude faster than magnetic diffusion and fluid convection, the only phenomena for magnetic transport seen in Hall magnetohydrodynamics. Such phenomena have been observed, for example, in pulsed plasmas and plasma opening switches.

- Hall physics may also explain the much smaller magnetic fields created through dynamo process – processes which may result in outgoing Hall electric jets in some stellar configurations.

- All results shown here are done for the simple case of electron-ion Hall plasmas. To express the problem in terms of dusty Hall plasmas, we note the following justification for the use of Hall physics:[1]

  - Dust particles, observed to be prevalent in stellar disks, is not magnetized due to dust-gas friction, but electrons and ions remain magnetized.

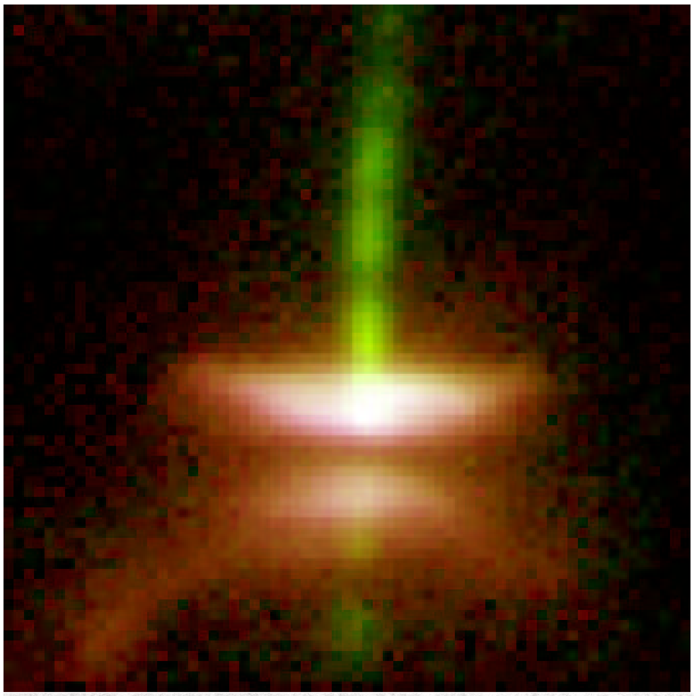
  - All electron-ion Hall physics can be substituted with dusty plasma by making following: $n \to n_d$, $e \to Z_d e$, $\eta \to \eta_i$, $V \to V_d$, $\eta_i = \frac{\nu_{ig} M_i c^2}{4\pi e^2 n_i} \times \frac{1}{(Z_d n_d / n_i)^2}$.
    * ν_{ig} is ion-gas collision frequency.
    * M_i is ion mass.
    * n_d, Z_d, V_d are dust number densities, charges, and velocities.

  - Dust inertial scale, for typical charge of dust $Z_d = -1$ and calculated densities of dust $n_d \sim 10^{-5}\text{cm}^{-3}$, are of order $D \sim 10$ AU, on scale of dimensions of problem.

[1] from L. Rudakov, "Dynamo and Electrical Jet in Hall Plasmas, Application to Astrophysics", http://arXiv.org/abs/astro-ph/0106003

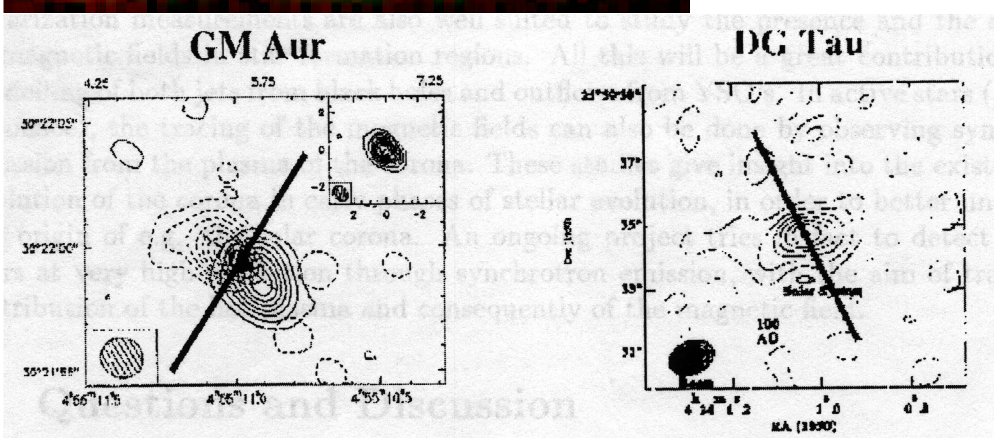
Justification

Possible role of Hall physics in jets about young stellar objects



Hubble Space Telescope shot of young stellar object HH30, taken from NASA archive.

Studies of the movement of substructure within the jets (calculated to have width of 50 AU), show a velocity five times larger than bulk velocity of jet (see M. Pestalozzi, "The Outflow-Disk Interaction in Young Stellar Objects," `http://arXiv:astro-ph/0007156` (2000)), as observed using Doppler shift measurements in radio (see Lada, C.J., and Fich, M., *ApJ* **459**, 638 (1996) for doppler measurements in radio of HH30).



Measurements of the polarization of the magnetic field in the jets of Herbig Haro objects GM Aur and DG Tau (see Tamura M., Hough G.H., Greaves J.S., Morino J.-I., Chrisostomou A., Holland W.S., and Momose M., *ApJ* **525**, 832 (1999)) imply toroidal magnetic field, **which may be due to presence of nonlinear Whistler drift modes, propagating as Burgers shock-wave.**

HALL DYNAMIC EQUATIONS

- Considering Hall plasma, in which *electrons* frozen into magnetic field and *ions* unfrozen in the liquid.

$$n\frac{\partial}{\partial t}\left(\frac{\mathbf{B}}{n}\right)+n\mathbf{v}_e\cdot\nabla\left(\frac{\mathbf{B}}{n}\right)=\mathbf{B}\cdot\nabla\mathbf{v}_e-\nabla\times(\eta\nabla\times\mathbf{B})$$

- Aassume that ions move with velocity $r\Omega(r,z)\mathbf{e}_\phi$. Also assume axisymmetric magnetic field given by $\mathbf{B}=B\mathbf{e}_\phi+\nabla\times(A\mathbf{e}_\phi)$ so we have following:

$$\begin{aligned}\frac{\partial\tilde{B}}{\partial\tilde{t}}&=\tilde{\eta}\left(\frac{1}{\tilde{r}}\left(r\frac{\partial\tilde{B}}{\partial\tilde{r}}\right)+\frac{\partial^2\tilde{B}}{\partial\tilde{z}^2}-\frac{\tilde{B}}{\tilde{r}^2}\right)-\frac{1}{\tilde{r}}\left(\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{z}}\frac{\partial}{\partial\tilde{z}}-\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{r}}\frac{\partial}{\partial\tilde{r}}\right)\tilde{\eta}-\\&\left(\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{z}}\frac{\partial}{\partial\tilde{r}}-\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{r}}\frac{\partial}{\partial\tilde{z}}\right)\tilde{\Omega}-\left(\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{z}}\frac{\partial}{\partial\tilde{r}}-\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{r}}\frac{\partial}{\partial\tilde{z}}\right)\frac{\tilde{B}}{\tilde{n}\tilde{r}}-\\&\left(\frac{\partial\left(\tilde{r}\tilde{A}\right)}{\partial\tilde{r}}\frac{\partial}{\partial\tilde{z}}-\frac{\partial\left(\tilde{r}\tilde{A}\right)}{\partial\tilde{z}}\frac{\partial}{\partial\tilde{r}}\right)\frac{1}{\tilde{n}\tilde{r}}\left(\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial\tilde{A}}{\partial\tilde{r}}+\frac{\partial^2\tilde{A}}{\partial\tilde{z}^2}-\frac{\tilde{A}}{\tilde{r}^2}\right)\right)\end{aligned}$$

$$\frac{\partial\tilde{A}}{\partial\tilde{t}}=\tilde{\eta}\left(\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial\tilde{A}}{\partial\tilde{r}}\right)+\frac{\partial^2\tilde{A}}{\partial\tilde{z}^2}-\frac{\tilde{A}}{\tilde{r}^2}\right)-\frac{1}{\tilde{n}\tilde{r}^2}\left(\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{z}}\frac{\partial}{\partial\tilde{r}}-\frac{\partial\left(\tilde{r}\tilde{B}\right)}{\partial\tilde{r}}\frac{\partial}{\partial\tilde{z}}\right)\tilde{r}\tilde{A}$$

- Normalizations are given by following: $\tilde{r}=r\omega_{pi}/c$, $\tilde{z}=z\omega_{pi}/c$, $\tilde{t}=t\omega_{ci}$, $\tilde{\Omega}=\Omega/\omega_{ci}$, $\tilde{B}=B/B_0$, $\tilde{A}=A/A_0=A/B_0\times(\omega_{pi}/c)$, $\tilde{\eta}=\nu_{ei}/\omega_{ce}$, $\tilde{n}=n/n_0$, $\omega_{pi}^2=4\pi n_0e^2/m_i$, $\omega_{pe}^2=4\pi n_0e^2/m_e$, $\omega_{ci}=eB_0/m_ic$, $\omega_{ce}=eB_0/m_ec$.

- **ignore gradients in plasma resistivity** η**; assume jet-like solutions** $A\equiv A(r,z-ut)$**,** $B\equiv B(r,z-ut)$**; only radial density dependence** $n\equiv n(r)$**. To get Grad-Shafranov-like solution**:

$$\begin{aligned}&r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r}\right)+\frac{\partial^2\Psi}{\partial z^2}=-\left(H(\Psi)-u\int_0^r r'n\left(r'\right)\,dr'\right)\frac{dH}{d\Psi}+nr^2\left(G(\Psi)-\Omega(r,z)\right)\\&rB(r,z-ut)=H(\Psi)-u\int_0^r r'n\left(r'\right)\,dr'\equiv\text{poloidal current}\\&rA(r,z-ut)=\Psi(r,z)\equiv\text{poloidal magnetic flux}\end{aligned}$$

- Results can be derived by force balance equation in frame comoving with velocity u, where $\mathbf{E}'=\mathbf{0}$, with bulk ion motion Ω.

HALL DYNAMO

- Consider simple model of Hall dynamo, with Hall plasma disk thickness $d \ll R$, (R being radius of disk), such that $\eta \gg d/R$ – allows us to ignore poloidal magnetic rotation $\delta A \ll B$.
- Simplest geometry:
  1. rigidly rotating disk $\Omega =$ constant sitting within motionless conducting Hall plasma (or rather length scale of Ω gradient is δ).
  2. Initial $B_z = B_0$ field threads Hall plasma conductor.
  3. $B = br$, $A = ar$ through conductor.
- Allows for following:

$$\frac{\partial b}{\partial t} = 2b\frac{\partial b}{\partial z} + \frac{\partial \Omega}{\partial z} + \eta\frac{\partial^2 b}{\partial z^2}$$

Which yields, since $\eta \to 0$ (large magnetic Reynold's number $R_m = v_A d/\eta$, $v_A = B_0/\sqrt{4\pi n M_i}$):

$$B_\phi \approx B_0 \left(R\Omega/v_A\right)^{1/2} \left(R/\omega_{pi} c\right)^{1/2}$$

Which is much smaller than MHD case of $B_\phi \approx B_0 R\Omega d/\eta \to \infty$ as $\eta \to 0$ (if we neglect Hall term $2b\partial b/\partial z$).

1. The first case denotes one where we can ignore the Hall term; in this instance, within the disk, $d^2B/dz^2 = 0$ (i.e. the MHD dynamo).

2. The second case is when the quantity $\Omega B_z < 0$, in which we have a steady-state Burgers-like shock (i.e. profile that goes as $\tanh z/\ell$), which has a toroidal amplitude at the upper and lower surfaces of $B_z \left(R\Omega/v_A\right)^{1/2} \times \left(R\omega_{pi}/c\right)^{1/2}$. The velocity of this shock front, before the equilibration shown in the cartoon, is $u = v_A \left(c/v_A \times \Omega/\omega_{pi}\right)^{1/2}$ for each limb of the shock front. The thickness of the front is η/v_A.

3. In a steady state, power dissipates through Joule heating within the central current layer, but the total resulting power is independent of resistivity. This power is equal to B_r^2 along the surface and $\frac{cB_\theta}{4\pi enR}$ (Hall velocity) times $B_\theta^2/4\pi$ (due to equal energy flux contributions from both sides).

4. The third case, occurs when $\Omega B_z > 0$. Here, toroidal magnetic flux flows outwards and we have a Burgers-like shock, with the aforementioned velocities and shock front thickness, **propagating out of the disk into an (assumed) Hall plasma column**.

These results were taken from L. Rudakov, "Dynamo and Electrical Jet in Hall Plasmas, Application to Astrophysics", `http://arXiv.org/abs/astro-ph/0106003`.

Electrical Jets in Hall Plasma

- Here, we assume following jet-like solutions $A \equiv A(r, z - ut)$, $B \equiv B(r, z - ut)$, or show how such solutions can arise, that propagate through a Hall plasma column.

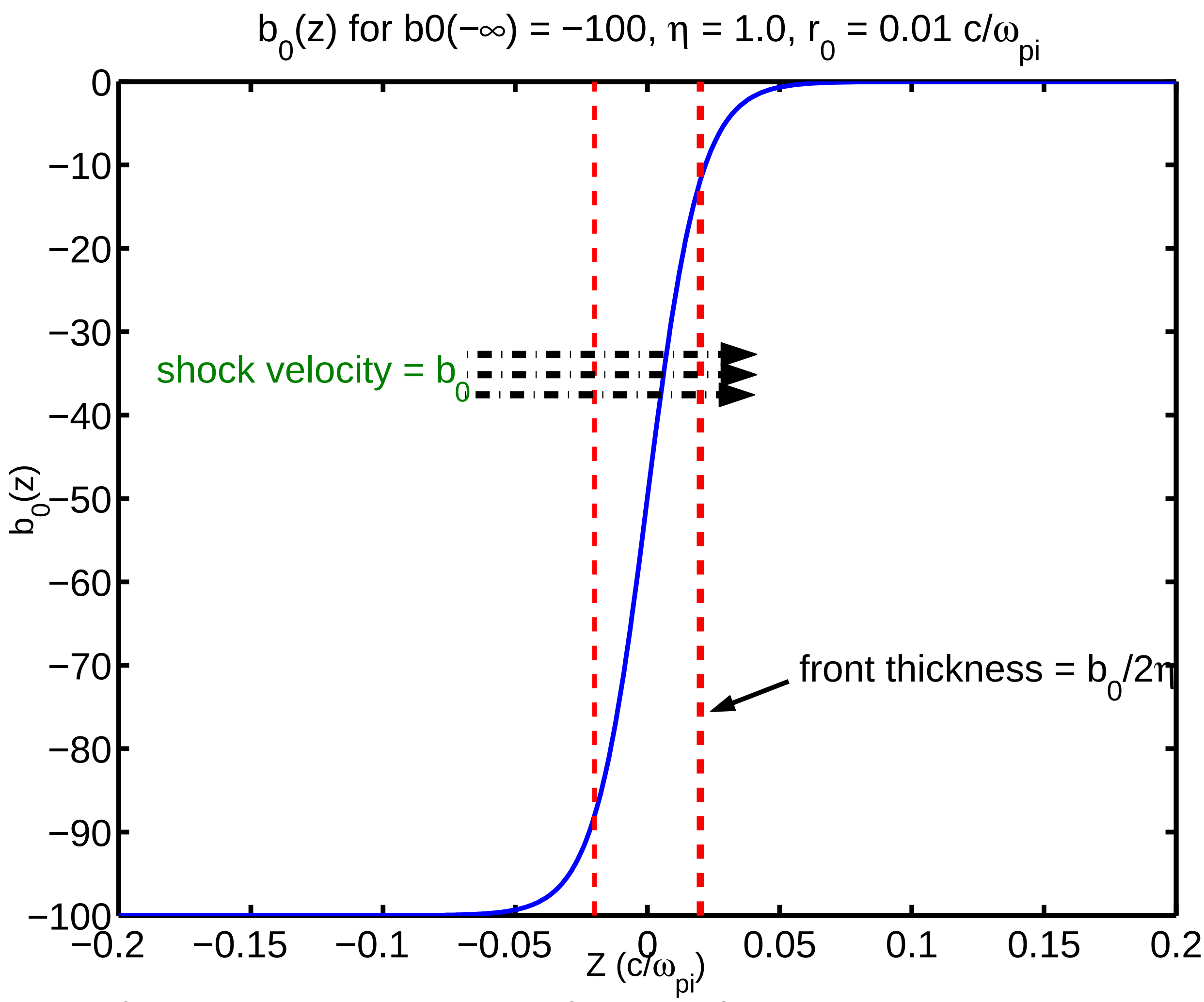
- We acquire solutions that are in some cases analogous to whistler drift modes (toroidal magnetic convection into bulk unmagnetized plasma)

  whistler modes – poloidal magnetic field rotation, or in other words possessing a three-component magnetic field.

  Some combination of the two linear Hall phenomena

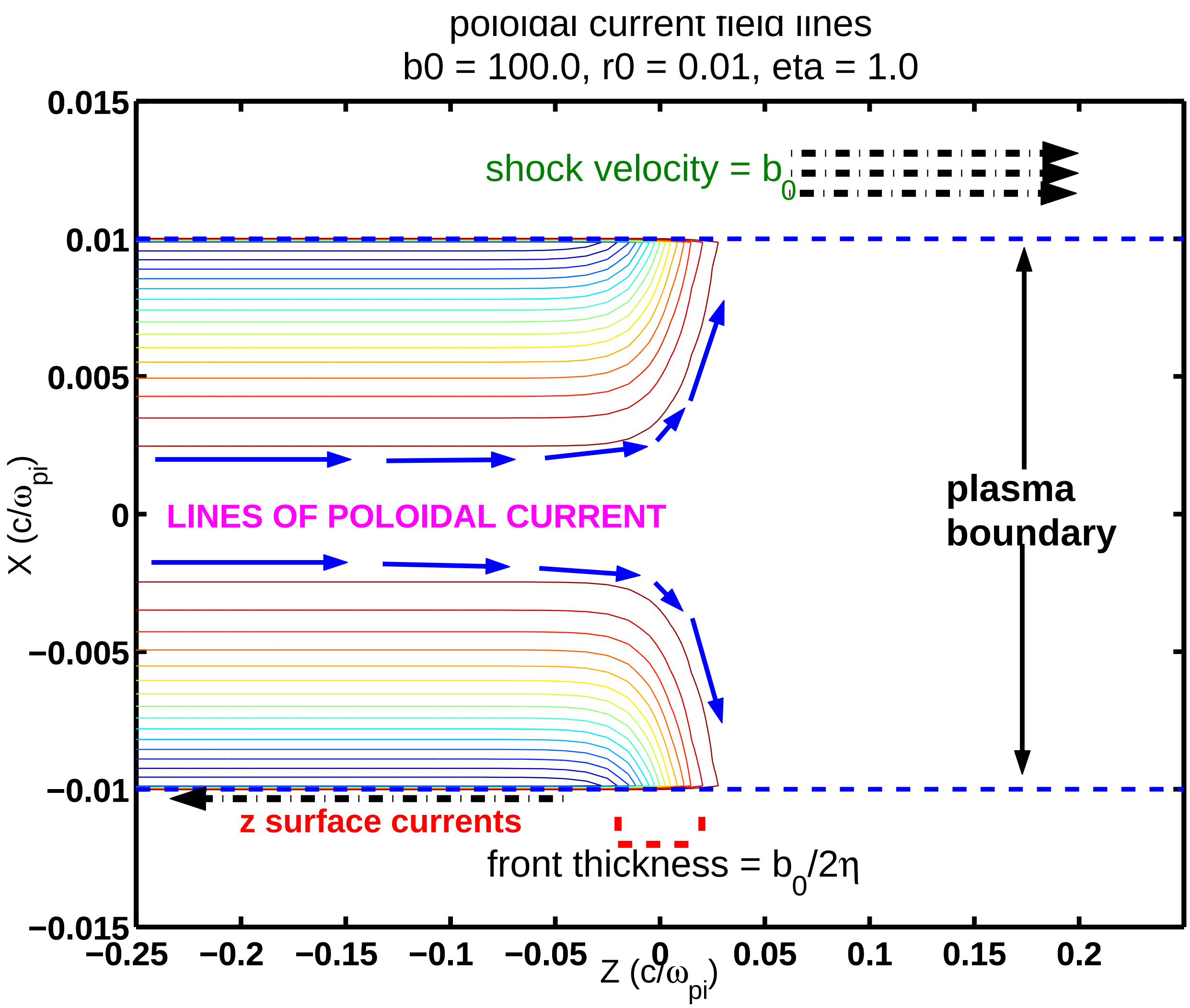
- All results represented in Hall normalized manner, with constant density within a plasma column, vacuum outside plasma column.

Electric Jet I: Resistive Nonlinear Shock Solution



Plot of the toroidal magnetic field profile in z within the plasma column (or specifically, the toroidal magnetic field is given by $B(r, z, t) = rb(r, z, t)$). We naturally expect that the "preshock" region has zero toroidal magnetic field. Here, in normalized units, $u = 100.0$, $b(-\infty) \rightarrow -100.0$, and the front thickness is $\delta = 0.02c/\omega_{pi}$. Here, we have large η and $A \ll B$ so that for $r < R$, $B(z, r, t) = \frac{1}{2} r b_0 \left(\tanh \left(\frac{b_0(z - b_0 t)}{2\eta} \right) - 1 \right)$, and for $r > R$, $B(z, r, t) = 0$.

Electric Jet I: Resistive Nonlinear Shock



Poloidal current lines within the Burgers-like nonlinear solution. Here, we see that a z current sheet develops on the Hall plasma column, that distributes current back into the (attached) accretion disk.

For our jet-like solution $(A, B) \equiv (A, B)(r) \cos k(z-ut)$ with $\eta \to 0$, solution given by following:

$$\Psi(r, z-ut) = rA(r, z-ut) =$$
$$\begin{cases} \alpha r J_1\left(r\sqrt{\kappa^2 - k^2}\right) \cos k\,(z-ut) + ur^2/\kappa & r < R \\ uR^2/\kappa & r > R \end{cases}$$
$$B(r, z-ut) = \kappa A(r, z-ut) - ur^2/\kappa =$$
$$\begin{cases} \kappa\alpha J_1\left(r\sqrt{\kappa^2 - k^2}\right) \cos k\,(z-ut) & r < R \\ 0 & r > R \end{cases}$$
$$J_1\left(R\sqrt{\kappa^2 - k^2}\right) = 0$$
$$\kappa^2 > k^2$$

With the following normalized toroidal surface current:

$$I_\phi(R, z) = \alpha\sqrt{\kappa^2 - k^2}\left(J_0\left(R\sqrt{\kappa^2 - k^2}\right) - J_2\left(R\sqrt{\kappa^2 - k^2}\right)\right) \times \cos k(z-ut) + 2u/\kappa$$

Simulation has $\alpha = 1.0$, $k = 1.0$, and $u = 100.0$, with κ chosen to be first zero of J_1 at plasma edge, $\kappa = \sqrt{k^2 + (3.873171/R)^2}$.

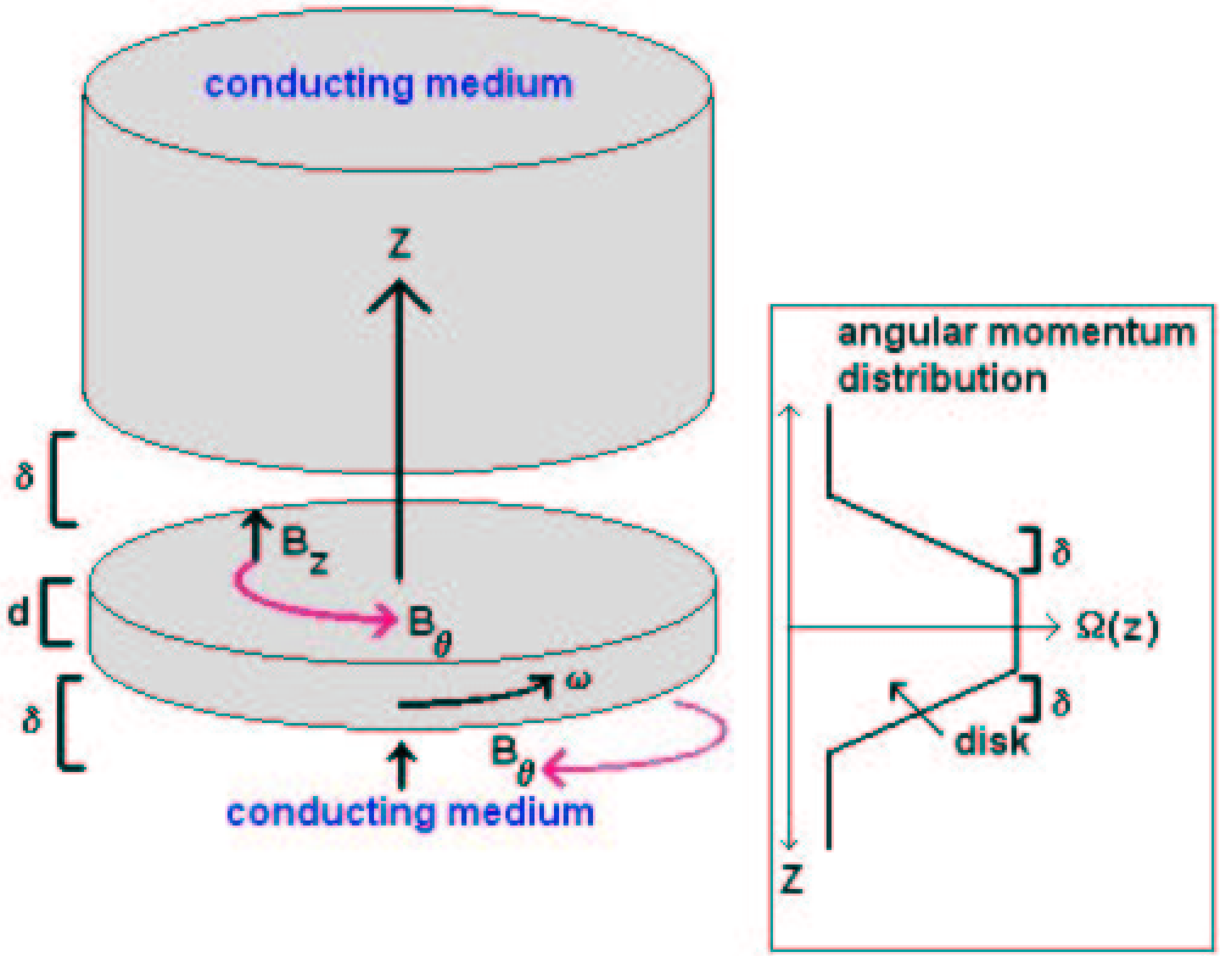
Conclusions and Further Research

- **Determine a larger variety of solutions, especially to the soliton-like solution and Grad-Shafranov like solution, in the Hall electrical jet, as well as more unstable solutions to determine in what manner these proposed Hall magnetic fields are created.**

- **In connecting electron-ion Hall plasma results to that of the dusty plasma, we need to take into account the fact that dust has a specific charge and mass distribution. One needs to develop this "kinetic" theory with dust to get better understanding.**

- **Analytic and simple numerical results may benchmark the performance of 3D numerical Hall simulation codes; likewise, Hall numerical simulations of astrophysical phenomena may suggest further directions of study.**

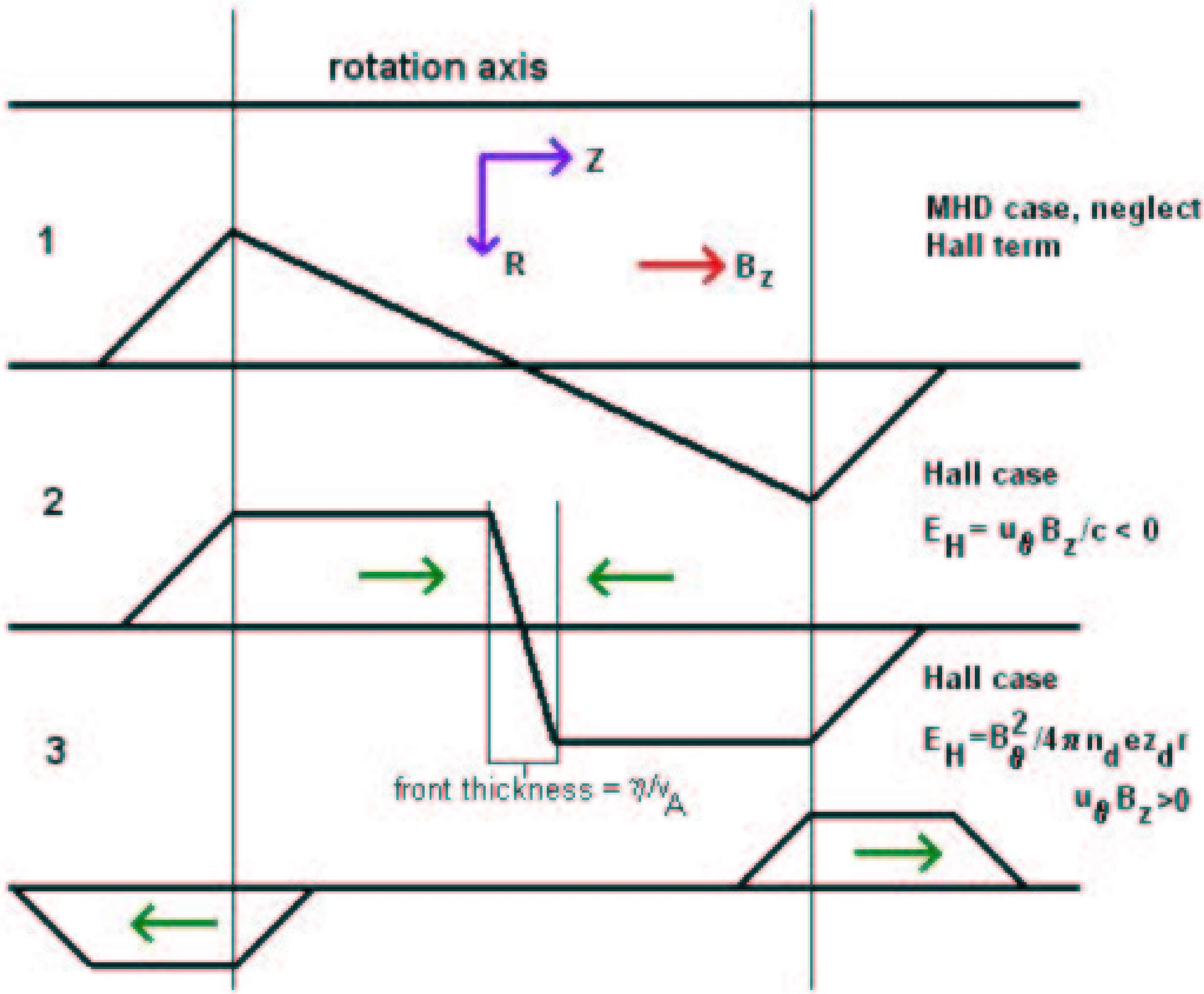
- **There may be other astrophysical phenomena in which, numerically and analytically, the Hall physics treatment may be more valid than standard MHD. Stellar accretion disks, analyzed here, is one. Dynamic instabilities in large interstellar molecular clouds may be another.**



Cartoon depicting the simplified Hall dynamo with finite (but small) resistivity. In the figure, we see that a rigidly rotating disk of thickness $d \ll R$ is rotating with frequency Ω, within a motionless plasma. The length scale for the rotational frequency gradient is δ, depicted on the graph to the right. The space between the "conductors" is a gap to depict more clearly the rotation differential.

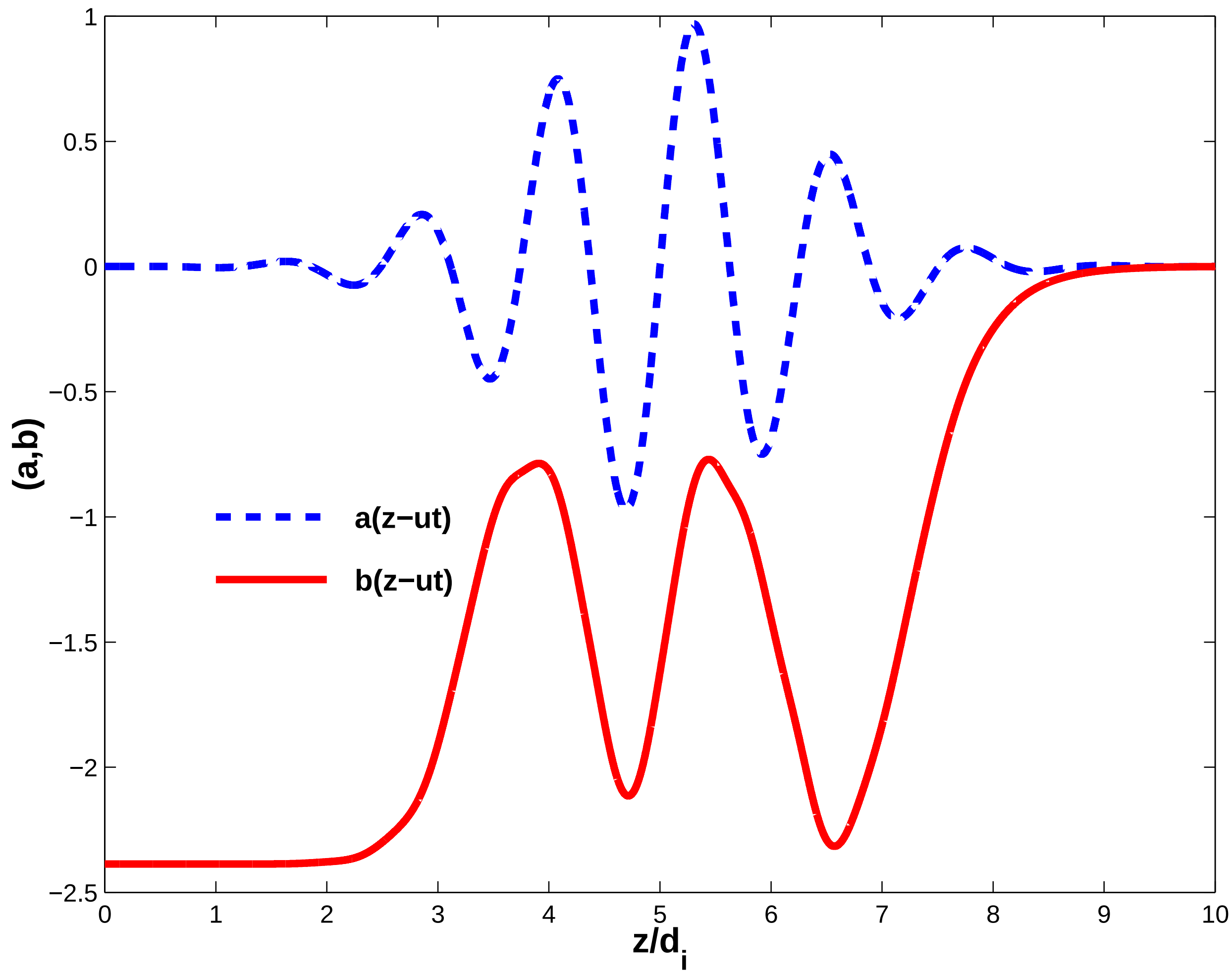
The entire system is originally threaded with a B_z field. Note that the rotation differential causes opposite signs of B_θ to appear, since from greater z, Ω changes from 0, whereas moving out of the disk through the bottom, Ω reverts to zero again.

In Hall limit, $B_\theta \approx B_0 (R\Omega/v_A)^{1/2} \times (R\omega_{pi}/c)^{1/2}$, where $v_A = B_0/\sqrt{4\pi n M_i}$.



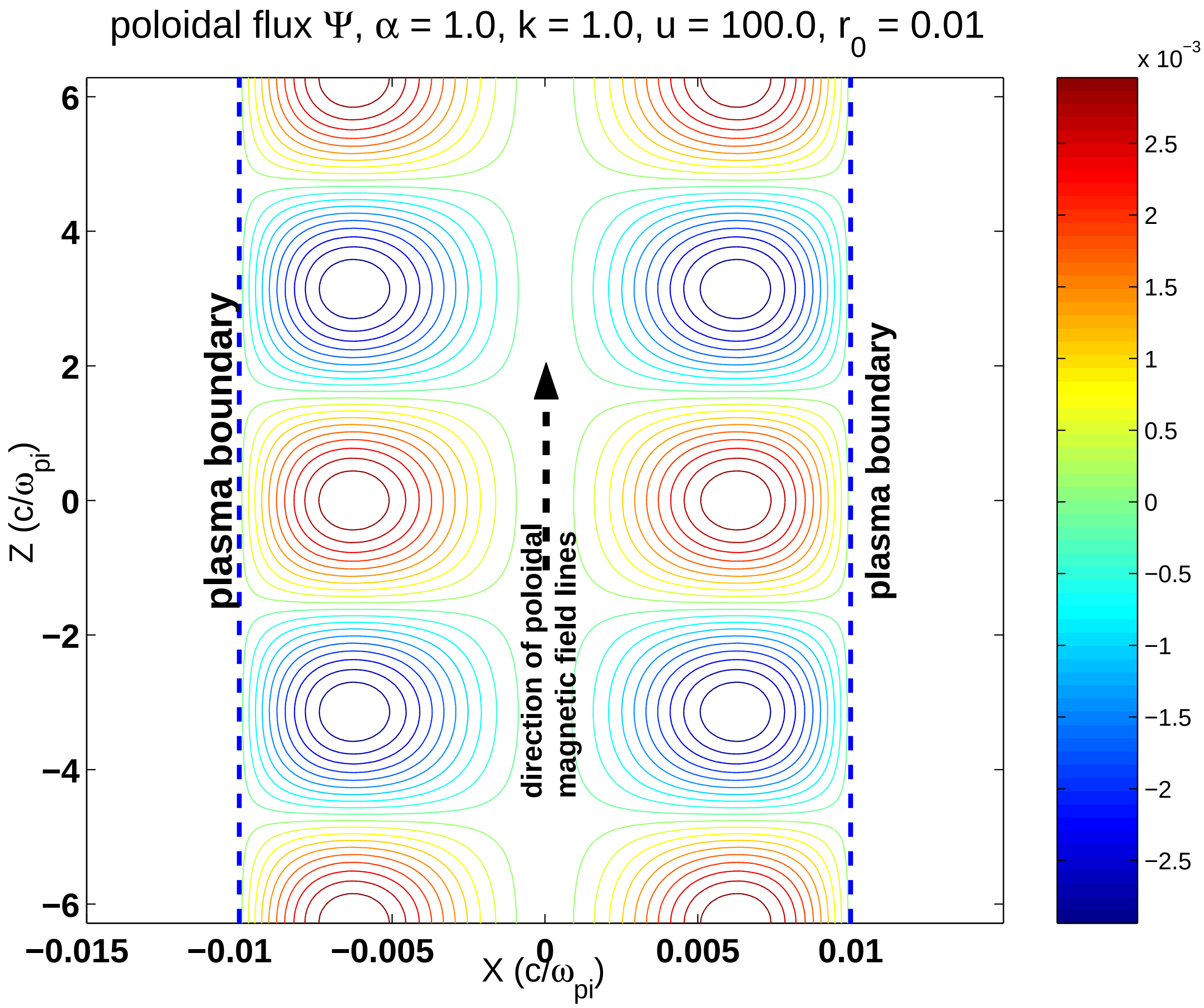
Depiction of toroidal magnetic field convection for three important cases, where we consider for simplicity a disk within which $d\Omega/dz = 0$. E_H denotes the value of the toroidal Hall electric field, where applicable. Green lines denote the toroidal magnetic flux flow, where applicable.

Electric Jet II: Soliton-Like Solution



Plot of a "soliton-like" solution for b (toroidal magnetic field) with a given specified perturbative a (toroidal vector potential). In normalized Hall units, $a(z) = e^{-z^2/3}\sin(5z)$, $u = 2.7356$, and $\lim_{b\to-\infty} b(z) = -2.3678$, and $\eta = 1$. "Shock-like" solutions, here denoting solutions in which the magnetic field approaches some limiting value at $\pm\infty$, are seemingly allowed only when $a \sim b \sim \eta$, and the specified a is localized.

Electric Jet III: Grad-Shafranov-Like Solution



Poloidal magnetic flux Ψ (and hence, poloidal magnetic field lines) for the Grad-Shafranov-like solution. Note that since the poloidal flux within the vacuum is constant, no poloidal magnetic fields exist within the vacuum. Note that the discontinuity in B_z results in ϕ current sheet along the Hall column surface.