The fundamental limitation on the number of possible quantum states in a bounded region—or, alternatively, on the number of bits that can be coded in a bounded region—is given by the Bekenstein Bound [1,2]. The Bekenstein Bound is a consequence of the basic postulates of quantum field theory. A derivation will not be given here, but in essentials the Bekenstein Bound is just another manifestation of the Heisenberg Uncertainty Principle.

If, as is standard, the information I is related to the number of possible states N by the equation $I = \log_2 N$, then the Bekenstein Bound on the amount of information coded within a sphere of radius R containing total energy E is

$$I \leq 2\pi ER/(\hbar c \ln 2) \tag{C.1}$$

or, expressing the energy in mass units of kilograms,

$$I \leq 2.57686 \times 10^{43} \left(\frac{M}{1 \text{ kilogram}} \right) \left(\frac{R}{1 \text{ meter}} \right) \text{ bits} \tag{C.2}$$

For example, a typical human being has a mass of less than 100 kilograms, and is less than 2 meters tall. (Thus such a human can be enclosed in a sphere of radius 1 meter.) Hence, we can let M equal 100 kg and R equal 1 meter in formula (C.2) obtaining

$$I_{human} \leq 2.57686 \times 10^{45} \text{ bits} \tag{C.3}$$

as an upper bound to the number of bits I_{human} that can be coded by any physical entity the size and mass of a human being.

Let me give an elementary *plausibility argument* for the Bekenstein Bound (C.1). This argument will be nonrigorous. (A completely rigorous proof would involve too much quantum field theory to be feasible in this book.) The Uncertainty Principle tells us that

$$\Delta P \Delta R \geq \hbar \tag{C.4}$$

Where ΔP is the ultimate limit in knowledge of the momentum and ΔR is the limit in knowledge of the position. (Alternatively, the inequality (C.4) expresses the minimum size of a phase space division.) Thus, if the total