# Homework #8

1. Consider the area/volume problem, and why, it is believed, that life would move away from planets and onto planetesimals. Take the earth, for instance. It has a radius of $6 \times 10^6$ meters.

   (a) What is the surface area and volume enclosed by Earth?

   (b) Now suppose you blew up the earth into 1 trillion $10^{12}$ pieces, each piece $10^{-12}$ of Earth's volume. What is the volume of each of these pieces? *Note, the total volume of Earth material, if Earth was exploded, would be substantially larger than the current volume of Earth – a big chunk of Earth's mass is compressed by millions of atmospheres of pressure.*

   (c) Now suppose each of these pieces is a sphere. Using the formula for a sphere, $\frac{4}{3}\pi R^3 = V$, where $R$ is the radius and $V$ is the volume, find the radius of each of these pieces.

   (d) Now calculate out the *total* surface area enclosed by these elements. How does this compare to the calculated surface area of Earth – that is, what is the ratio between the total new surface area and the old surface area.

   (e) Suppose we wish to construct a habitat with a period of 24 hours and an acceleration of 10 m/s$^2$ – Earth gravity. What is the radius of rotation of this object? Remember to calculate $\omega$ for this object, and that $T = 2\pi/\omega$, where $T$ is the rotation period and $\omega$ is the rotational frequency.

2. Now calculate out the sort of "fundamental" shortest scales in our Universe. It is expected that these quantities depend on $G = 6.673 \times 10^{-11}$ kg$^{-1}$ m$^3$ s$^{-2}$ being the gravitational constant, $h = 6.63 \times 10^{-34}$Joule-seconds being Planck's constant, and $c = 3 \times 10^8$ m/s being the speed of light.

   Using the prescription given in class, letting $G^\alpha h^\beta c^\gamma$ equal some dimensional quantity, and noting that: 1) $G \equiv M^{-1}L^3T^{-2}$, 2) $h \equiv ML^2T^{-1}$, and 3) $c \equiv LT^{-1}$:

   (a) Calculate out something with units of mass $M^1L^0T^0$, and give its value.

   (b) Calculate out something with units of length $M^0L^1T^0$, and give its value.

   (c) Calculate out something with units of time $M^0L^0T^1$, and give its value.

3. Now consider the habitability of a Dyson sphere.

   (a) For a Dyson sphere of radius equal to 1 AU ($1.5 \times 10^{11}$ meters), what level of population can it support, assuming that Earth, with a radius of $6 \times 10^6$ meters, can support 10 billion people? To solve this problem, you need to find the surface areas of Earth and the Dyson sphere.

   (b) The volume of a Dyson sphere is approximately $V = 4\pi R^2 \Delta R$, where $\Delta R$ is the thickness of the shell. If a Dyson sphere has radius of 1 AU and a thickness of 30 centimeters.

      i. What is its volume?

      ii. Assume the sphere has a mass of Jupiter, so that $M_{\text{Dyson}} = 1.96 \times 10^{27}$ kg, what is the *mass density* of the Dyson sphere? How does this compare to water – what is the ratio of the Dyson sphere mass density to that that of water, with density 1000 kg/m$^3$?

4. **Extra Credit II – 36 points** The emission of radiation from any body is a function its mass $M$, radius $R$, and Planck's constant $h = 6.63 \times 10^{-34}$ J-seconds.

   (a) Construct something with units of power (units of $ML^2T^{-1}$) from $M$, $R$, and $h$.

(b) The radius of a black hole with mass $M$ is approximately $R \sim GM/c^2$. Now what is the power that leaves its black hole as a function of black hole mass?

(c) Now given that the power output corresponds to the mass loss rate, how would I calculate the lifetime $\tau$ of the black hole, assuming that $P = Mc^2/\tau$, where $P$ is power?

(d) What is the lifetime, using the above formula, for a 1 solar mass black hole, $M = 2 \times 10^{30}$ kg? How about for a $10^9$ solar mass black hole?