Homework #7

1. Consider the various types of Kardashev level civilizations that might currently exist – or at least, if we survive into the far future, we will probably end up as one!

   (a) The sun has a luminosity of 10^{26} Watts. A Kardashev I civilization uses up all the energy falling on the planet's surface. Consider Earth to be a circular "detector" with radius $R_{\oplus} = 6 \times 10^6$ m, so $A_{\text{detect}} = \pi R_{\oplus}^2$. The earth is situated 1 AU, or $D = 1.5 \times 10^{11}$ meters from the sun. Determine the energy output of a Kardashev I civilization at Earth orbit.
   The earth has radius of $R_{\oplus} = 1.2 \times 10^6$ m, so the total amount of energy falling on the earth:

   $$L_{\oplus} = \frac{\pi R_{\oplus}^2}{4\pi D^2} L_{\odot} = \frac{1}{4}\left(\frac{6 \times 10^6}{1.5 \times 10^{11}}\right)^2 \left(4 \times 10^{26}\right) = 6.4 \times 10^{17} \text{ W}$$

   (b) A Kardashev III civilization will harness all the energy within a galaxy. Suppose, on average, the luminosity of stars are $L_\star = 0.1 L_\odot$ (the average luminosity of stars is 10% the solar luminosity), and there are 10^{11} stars in our galaxy. What is the energy output of a Kardashev III civilization?
   The total power available to a Kardashev III civilization is given by:

   $$P_{III} = 0.1 \times 10^{11} \times L_\odot = 10^{10} \times 4.26 \times 10^{26} = 4.26 \times 10^{36} \text{ W}$$

   (c) Supposing the per-capita usage of energy is 1 gigawatt (this takes into account all types of industry and transportation). Based on this alone, how many "people" could a Kardashev III civilization support (on energy alone)?
   Supposing the per-capita usage of power was 10^9 Watts per person on average, then the number of people that could be supported is $N = 4.26 \times 10^{36}/10^9 = 4.26 \times 10^{27}$ people on power alone.

   (d) Instead of supposing that the energy for a Kardashev III civilization comes from stars, suppose it comes from gravitational collapse.

      i. From the equation $E \approx Mc^2$, estimate the energy available to a Kardashev III civilization (from the collapse of all the matter into a black hole); take $M = 10^{11}$ $M_\odot = 2 \times 10^{41}$ kg and $c = 3 \times 10^8$ m/s.
      Using the $E = mc^2$ formula, with the total mass of the galaxy, the total energy available is $E = (2 \times 10^{41}) \times (3 \times 10^8)^2 = 1.8 \times 10^{58}$ Joules available from gravitational collapse into a black hole.

      ii. From your estimate of power consumption of a Kardashev III civilization, estimate the lifetime of such a civilization in years. How does this compare to the lifetime of the sun? This is approximately equal to the lifetime of the dimmest stars – but the energy usage is utterly inconceivable.
      Given the power consumption rate and the total energy available, one gets that the lifetime of this Kardashev III civilization:

      $$T_{III} = \frac{1.8 \times 10^{58}}{4.26 \times 10^{36}} = 4.23 \times 10^{21} \text{ s} = 1.34 \times 10^{14} \text{ years}$$

2. The upper limits on information processing with atoms as specific elements is limited by light speed as well as the size of atoms – although I put the thermodynamic noise as an upper limit, it is really the smaller of the two.

(a) Given the speed of light and the fact that atoms are separated by a distance of 10^{-10} meters, what is the fastest possible "switch" rate of atom-based information elements?

The speed of light is $c = 3 \times 10^8$ m/s. The electronic (or other) elements are 10^{-10} meters apart. Therefore the switch time $\tau_{\text{switch}} = 10^{-10}/\left(3 \times 10^8\right) = 3.33 \times 10^{-19}$ seconds. For your own curiosity, the fastest discrete chemical reactions (molecule-molecule reactions) take place over 10^{-12} seconds.

(b) The average energy per calculation of an atomic-sized element is roughly the energy that can be placed within an atom or molecule. This is roughly 2×10^{-20} Joules. How many Joules per second are used up per atom each second, given the number of calculations to be done per second – this is the power per atom.

Each "atom" can perform 3×10^{18} calculations per second. However, each calculation uses up 2×10^{-20} J. Thus, the power per atom (Joules/second) is $\epsilon_{\text{atom}} = 3 \times 10^{18} \times 2 \times 10^{-20} = 6 \times 10^{-2}$ W/atom.

(c) Now suppose the atoms are spread over a surface. There are 10^{20} atoms per square meter. Given the power per atom, what is the power per square meter of computing surface? This corresponds to a surface temperature of 3 million Kelvins!

Given the fact that there are 10^{20} atoms/m^2, the total power flux:

$$F_P = \frac{10^{20}\text{ atoms}}{1\text{ m}^2} \times \frac{6 \times 10^{-2}\text{ W}}{1\text{ atom}} = 6 \times 10^{18}\text{ W/m}^2$$

Which results in a surface temperature of 3.2×10^6 Kelvins.