Homework #7

1. Consider the various types of Kardashev level civilizations that might currently exist – or at least, if we survive into the far future, we will probably end up as one!

   (a) The sun has a luminosity of 10^{26} Watts. A Kardashev I civilization uses up all the energy falling on the planet's surface. Consider Earth to be a circular "detector" with radius $R_{\oplus} = 1.2 \times 10^6$ m, so $A_{\text{detect}} = \pi R_{\oplus}^2$. The earth is situated 1 AU, or $D = 1.5 \times 10^{11}$ meters from the sun. Determine the energy output of a Kardashev I civilization at Earth orbit.

   (b) A Kardashev III civilization will harness all the energy within a galaxy. Suppose, on average, the luminosity of stars are $L_\star = 0.1 L_\odot$ (the average luminosity of stars is 10% the solar luminosity), and there are 10^{11} stars in our galaxy. What is the energy output of a Kardashev III civilization?

   (c) Supposing the per-capita usage of energy is 1 gigawatt (this takes into account all types of industry and transportation). Based on this alone, how many "people" could a Kardashev III civilization support (on energy alone)?

   (d) Instead of supposing that the energy for a Kardashev III civilization comes from stars, suppose it comes from gravitational collapse.

      i. From the equation $E \approx Mc^2$, estimate the energy available to a Kardashev III civilization (from the collapse of all the matter into a black hole); take $M = 10^{11}$ $M_\odot = 2 \times 10^{41}$ kg and $c = 3 \times 10^8$ m/s.

      ii. From your estimate of power consumption of a Kardashev III civilization, estimate the lifetime of such a civilization in years. How does this compare to the lifetime of the sun? This is approximately equal to the lifetime of the dimmest stars – but the energy usage is utterly inconceivable.

2. The upper limits on information processing with atoms as specific elements is limited by light speed as well as the size of atoms – although I put the thermodynamic noise as an upper limit, it is really the smaller of the two.

   (a) Given the speed of light and the fact that atoms are separated by a distance of 10^{-10} meters, what is the fastest possible "switch" rate of atom-based information elements?

   (b) The average energy per calculation of an atomic-sized element is roughly the energy that can be placed within an atom or molecule. This is roughly 2×10^{-20} Joules. How many Joules per second are used up per atom each second, given the number of calculations to be done per second – this is the power per atom.

   (c) Now suppose the atoms are spread over a surface. There are 10^{20} atoms per square meter. Given the power per atom, what is the power per square meter of computing surface? This corresponds to a surface temperature of 3 million Kelvins!