Homework #6

1. The human eye is a lens with a diameter of 1 cm. The wavelength of light is typically 500 nm (5×10^{-7} m). What is the angular resolution, in degrees, of the human eye to visible light?
   The angular resolution in radians:

   $$\theta = \frac{\lambda}{D} = \frac{5 \times 10^{-7}}{0.01} = 5 \times 10^{-5} \text{ radians}$$

   To convert into degrees, note that there are 180 degrees in π radians. $\theta = 5 \times 10^{-5} \times (180/\pi) = 2.86 \times 10^{-3}$ degrees.

2. Sometimes astronomers want to know if an object resolved through their telescope is a "point" or a "disk." An object, such as a star, is considered to be a "disk" if it can just be resolved by a telescope – that is, the angular size of the object (calculated as $\theta = D_{\text{obj}}/R$, where D_{obj} is the size of the object and R is the distance to the object) is equal to the angular resolution of your device $\theta_{\text{min}} = \lambda/D$, where λ is the wavelength of light and D is the diameter of the telescope. Calculate the threshold distance using the formula:

   $$\frac{\lambda}{D} = \frac{D_{\text{obj}}}{R}$$

   (a) The sun is 1.4×10^9 meters across. Assume it emits light of only 500 nm. How far away do we have to go from the sun so that it appears as a "disk," given that our eyes are 1 cm in diameter?
   From the above formula, we need to determine R while knowing the other variables:

   $$R = \frac{D_{\text{obj}}}{\lambda} D = \frac{1.4 \times 10^9}{5 \times 10^{-7}} \times 0.01 = 2.8 \times 10^{13} \text{ m}$$

   This is 186.7 AU away.

   (b) Now repeat the above with a telescope with diameter of 1 meter. How far away can we observe the sun with 500 nm radiation for it be seen as a disk?
   Since the diameter of the observing apparatus is 100 times larger, therefore one can see the sun as a disk 2.8×10^{15} m away, or approximately 0.28 light-years.

   (c) The closest star is Proxima Centauri, which is 4.3 light-years away (remember, a light-year is 10^{16} meters), and has a diameter of 5×10^8 meters. What is the wavelength of light required for it to be seen as a disk?
   The diameter of the telescope is still 1 meter. The distance to Proxima Centauri, $R = 4.3 \times 10^{16}$ m. Now we solve for the wavelength of light.

   $$\lambda = \frac{D_{\text{obj}}}{R} D = \frac{5 \times 10^8}{4.3 \times 10^{16}} \times 1 = 1.16 \times 10^{-8} \text{ meters}$$

   This corresponds to light in the ultraviolet.

(d) The best radio telescope configurations achieve a resolution of 10^{-4} arc-seconds for 5 centimeter radiation. Will they be able to resolve a planet as large as Earth (with diameter $D = 1.2 \times 10^7$ meters) in the Proxima Centauri system – that is, see the planet as a "disk" rather than a "point"?

The angular size of a planet orbiting around Proxima Centauri, as seen from Earth a distance of 4.3×10^{16} m away, is:

$$\theta = \frac{1.2 \times 10^7}{4.3 \times 10^{16}} = 2.791 \times 10^{-10} \text{ radians}$$

To convert into arcseconds, note that there are 3600 arcseconds in a degree, and 180 degrees in π radians, therefore $\theta = 2.791 \times 10^{-10} \times 180/\pi \times 3600 = 5.76 \times 10^{-5}$ arcseconds. So not yet – these telescopes cannot resolve this "planet".

3. You have a typical ethernet connection operating at 10 megabits per second. A person can be described by 10^{18} bits of information. How long would it take to transport somebody with the bandwidth of an ethernet connection?
Just divide the bits over the bits per second, so the time $T = 10^{18}/10^7 = 10^{11}$ seconds = 3170 years. Plus, current ethernet connections are prone to frequent hangups!

4. **extra credit – 30 points** What frequency of light must one use to carry this person, if we use photons made from the person (recall that Planck's constant is $h = 6.626 \times 10^{-34}$ Joule-seconds, and the energy of a single photon of frequency ν is $E = h\nu$. Furthermore, $E = mc^2$, where m is the person's mass)? You may use your mass in kilograms – look on the web or book to find a pounds-kilograms conversion.
I will have a write-up later on Monday.