Homework #4

1. Consider again the problem of radiation due to the parent star.

   (a) The luminosity of the sun is $L_\odot = 4 \times 10^{26}$ W (1 Watt = 1 Joule/second). The earth is located at a distance $R_♁ = 1.5 \times 10^8$ m. How many watts per square meter fall on an object at Earth's distance from the sun (recall that the area of a sphere of radius R is $A = 4\pi R^2$)?
   Note that the correct distance is 1.5×10^{11} ***m – if you used the distance I gave, I will not penalize***. Using the formula for flux:

   $$F = \frac{L_\odot}{4\pi R_♁^2} = \frac{4 \times 10^{26}}{4\pi \left(1.5 \times 10^{11}\right)^2} = 1.41 \times 10^3 \text{ W/m}^2$$

   Of course, if you used the distance I gave, then your flux is larger by a factor of 10^6.

   (b) Now replace this distance $R_♁$ with 10 pc, so that $R = 10$ pc $\times$ (3.26 ly/pc) $\times$ (10^{16} m/ly). How many watts per square meter of radiation from the sun is there 10 pc from the sun?
   Using the formula, with the distance $R = 3.26 \times 10^{17}$ m:

   $$F = \frac{4 \times 10^{26}}{4\pi \left(3.26 \times 10^{16}\right)^2} = 3.00 \times 10^{-10} \text{ W/m}^2$$

   The collecting area of our eyes are 1 cm^2. The average energy of a photon reaching our eyes is approximately 4×10^{-19} J. So there are approximately 800 photons/second reaching our eyes.

   (c) Hart's model gives a CHZ at a radius from the sun of 0.97 AU $< R <$ 1.03 AU. Given that the power flux F goes as $1/R^2$, where R is the distance from the sun, (the power flux is 1/4 as large for an object 2 times as far from a star), what is the ratio of the power flux between the inner and outer edges of the CHZ (i.e., $F_{\text{inner}}/F_{\text{outer}}$)?
   From the proportionality relation given in the above, $F \propto R^{-2}$, this gives:

   $$\frac{F_{\text{inner}}}{F_{\text{outer}}} = \left(\frac{R_{\text{outer}}}{R_{\text{inner}}}\right)^2 = \left(1.03/0.98\right)^2 = 1.13$$

   The inner region of the CHZ is 13% brighter than the outer region of the CHZ.

   (d) Recall that the power flux, which you calculated in the above goes as $1/R^2$ (the power flux is 1/4 as large for an object 2 times as far from a star) and goes as L (the power flux at a given radius R from the star *doubles* as the luminosity of the star doubles). From these results, and given the fact that Jupiter is 5.2 AU away, how much brighter would the sun have to be in order that the power flux at Jupiter to equal the power flux from the sun at Earth is today?
   If we keep the solar luminosity *constant*, then the flux around Jupiter relative to that around Earth:

   $$\frac{F_♃}{F_♁} = \frac{R_♁^2}{R_♃^2} = \frac{1}{5.2^2}$$

   Thus, for the flux received at Jupiter to equal the flux from Earth, the sun would have to be $5.2^2 = 27.04$ times brighter.

2. Dimensional analysis again.

(a) I want you to figure out, to an order of magnitude, how much energy was liberated from the sun in its gravitational collapse. The only things that this energy depends on are G, the gravitational constant, which has dimensions of $L^3M^{-1}T^{-2}$ (where T is a time), M, the mass of the sun (has units of M a mass), and R, the current radius of the sun (has units of L a length). Construct a quantity from G, M, and R that has units of energy in the following way:

$$E = G^{\alpha}M^{\beta}R^{\gamma}$$

Recall that energy has units of ML^2T^{-2}.

Recall the prescription for constructing dimensions from other dimensions as shown in the above. Energy has units of ML^2T^{-2}.

$$M^1L^2T^{-2} = \left(M^{-1}L^3T^{-2}\right)^{\alpha} M^{\beta}L^{\gamma} = M^{-\alpha+\beta}L^{3\alpha+\gamma}T^{-2\alpha}$$

Matching the exponents results in three equations for three unknowns:

$$1 = -\alpha + \beta$$
$$2 = 3\alpha + \gamma$$
$$-2 = -2\alpha$$

Solving the above, we have that $\alpha = 1$, $\beta = 2$, and $\gamma = -1$. The potential energy of the earth is of the order of:

$$E = \frac{GM^2}{R}$$

(b) From the above expression, and given that $G = 6.673 \times 10^{-11}$ kg m^3 s^{-2}, $M_{\odot} = 2 \times 10^{30}$ kg, and $R_{\odot} = 7 \times 10^8$ m, calculate out the energy of the sun liberated from gravitational collapse, in Joules (recall 1 Joules = 1 kg m^2 s^{-2}).

Substitution of the above parameters into the equation determined in (1a):

$$E = \frac{\left(6.673 \times 10^{-11}\right)\left(2 \times 10^{30}\right)^2}{7 \times 10^8} = 3.81 \times 10^{41}\ \text{J}$$

This is about three orders of magnitude smaller than the total energy available from nuclear fusion.

(c) Now given the sun's luminosity $L_{\odot} = 4 \times 10^{26}$ W, calculate out the lifetime of the sun if the only source of energy is through gravitational collapse. Express your answer in millions of years.

The sun's lifetime, given the energy and its luminosity:

$$T = \frac{E_{\text{grav}}}{L_{\odot}} = \frac{3.81 \times 10^{41}}{4 \times 10^{26}} = 9.53 \times 10^{14}\ \text{s} = 30.2 \text{ million years}$$