Homework #3

1. Planets do not move around their parent star while the star remains motionless; instead a star and its planet move around a common center of mass. Suppose that a star has mass M and a planet has mass m, and that the star is much more massive than the planet (mathematically represented as $M \gg m$). Now given that the planet orbits around the center of mass at a distance R_{planet}, the star will orbit around the center of mass at a distance m/M smaller (that is, at a distance of $m/M \times R_{\text{planet}}$. Furthermore, a planet has a certain *orbital velocity* v_{planet} about the center of mass. The velocity of the star about its center of mass is $m/M \times v_{\text{planet}}$.

   (a) The sun is 330,000 times more massive than the earth ($M/m = 330,000$). The earth orbits at a distance of 1.5×10^8 km about the center of mass. What is the distance of the sun to the center of mass? The radius of the sun is 7×10^5 km. Is this distance smaller or larger than the sun's radius? Now repeat for Jupiter, whose mass is 1000 times smaller than the sun and whose distance from the sun is 5.2 AU. What is the wobble of the sun about its center of mass due to Jupiter?
   Using the fulcrum argument given above, the motion of the sun about the center of mass:

   $$R_\odot = \frac{M_{\text{planet}}}{M_\odot} R_{\text{planet}}$$

   Where R_{planet} is the distance of the planet to the center of mass. Therefore the wobble distance of the sun due to Earth, $R_{\odot-♁}$:

   $$R_{\odot-♁} = \frac{1.5 \times 10^8}{330,000} = 455 \text{ km} < R_\odot$$

   The wobble of the sun due to Jupiter, $R_{\odot-♃}$, is given by:

   $$R_{\odot-♃} = \frac{5.2 \times 1.5 \times 10^8}{1000} = 780,000 \text{ km} > R_\odot$$

   (b) The orbital velocity of the earth in its orbit is 30 km/s. What is the corresponding velocity of the sun about its center of mass? The orbital velocity of Jupiter is 13.2 km/s. What is the corresponding velocity of the sun about the sun-Jupiter center of mass?
   Using the fulcrum argument for velocities, we find that $v_{\odot-♁}$, the velocity of the sun about the sun-Earth center of mass:

   $$v_{\odot-♁} = \frac{M_♁}{M_\odot} v_♁ = \frac{30,000 \text{ m/s}}{330,000} = 9.09 \text{ cm/s}$$

   Likewise, the $v_{\odot-♃}$, the velocity of the sun about the sun-Jupiter center of mass:

   $$v_{\odot-♃} = \frac{M_♃}{M_\odot} v_♃ = \frac{13,200 \text{ m/s}}{1000} = 13.2 \text{ m/s}$$

   (c) Given the above velocities and orbital radii of Earth and Jupiter, determine their orbital periods. Remember that the circumference of a circle is $2\pi R$, where $\pi \approx 3.1415$.
   Earth and Jupiter move in circular orbits about their respective sun-planet centers of mass. The circumference of a circle $C = 2\pi R$. Thus, the time period of their orbits is given by, where R is the orbital radius and v is the orbital velocity:

   $$T = \frac{2\pi R}{v}$$

   Based on this, the orbital period of Earth, $P_♁$:

   $$P_♁ = \frac{2\pi R_♁}{v_♁} = \frac{2\pi \times \left(1.5 \times 10^8 \text{ km}\right)}{30 \text{ km/s}} = 3.142 \times 10^7 \text{ s} = 1.00 \text{ yr}$$

   And for Jupiter, $P_♃$:

   $$P_♃ = \frac{2\pi R_♃}{v_♃} = \frac{2\pi \times \left(5.2 \times 1.5 \times 10^8 \text{ km}\right)}{13.2 \text{ km/s}} = 3.713 \times 10^8 \text{ s} = 11.7 \text{ yr}$$

(d) Now calculate the period of the sun's wobble due to Earth and due to Jupiter. You should find that the wobble period due to Earth is equal to Earth's orbital period, and so for Jupiter.
Here you already have the velocities and orbital radii of the sun's wobble around the sun-Jupiter and sun-Earth centers of mass. The sun-Jupiter wobble period $P_{\odot-♃}$:

$$P_{\odot-♃} = \frac{2\pi R_{\odot-♃}}{v_{\odot-♃}} = 11.7\text{ yr}$$

and sun-Earth wobble period $P_{\odot-♁}$:

$$P_{\odot-♁} = \frac{2\pi R_{\odot-♁}}{v_{\odot-♁}} = 1.00\text{ yr}$$

The same period as the planets' orbital periods.

You should find relatively miniscule velocities and *proper motions* of the parent star due to their planets. Also, the period of the stars' wobble is the same as the orbital period of the planets themselves, making planetary detections even more difficult.

2. Now consider the brightness of planets relative to their parent star. Planets are like lenses always facing their sun, with surface area πR^2, where R is the planet radius. Assume that these planets reflect *all* the light that falls on them. To get the planet luminosity, use the formula for the luminosity that enters a detector, $L_{\text{detector}} = \frac{L_{\text{source}}}{4\pi D^2} \times A_{\text{detector}}$, where D is the distance between detector and source and A_{detector} is the area of the detector.

(a) The luminosity of the sun is $L_\odot = 4 \times 10^{26}$ W. The earth has a radius of 6000 km, and is 1.5×10^8 km from the sun. What is the earth's luminosity?
Using the formula given above, the earth's luminosity $L_{♁}$, assuming perfect reflectivity:

$$L_{♁} = \frac{4 \times 10^{26}}{4\pi \left(1.5 \times 10^8\right)^2} \times \pi \left(6000\right)^2 = 1.6 \times 10^{17}\text{ W}$$

(b) Jupiter has a radius of 70,000 km and is 5.2 AU from the sun. What is its luminosity?
Jupiter's luminosity, $L_{♃}$:

$$L_{♃} = \frac{4 \times 10^{26}}{4\pi \left(5.2 \times 1.5 \times 10^8\right)^2} \times \pi \left(70000\right)^2 = 8.05 \times 10^{17}\text{ W}$$

(c) What is the ratio of Earth's luminosity to the sun's (i.e., $L_{♁}/L_\odot$).
Here divide (1a) by the sun's luminosity:

$$\frac{L_{♁}}{L_\odot} = \frac{1.6 \times 10^{17}}{4 \times 10^{26}} = 4 \times 10^{-10}$$

This will give you an idea of how difficult it is to resolve planets around their parent stars. Also, planets do not reflect all the light from their parent stars, so their luminosities are less than calculated above.