Homework #2

1. The sun has mass $M_\odot = 2 \times 10^{30}$ kg, and it has a total power output of $L_\odot = 4 \times 10^{26}$ W. In its lifetime on the *main sequence* (defined as that period of time when a star burns only hydrogen within the core, rather than in a shell surrounding the core), the sun will "burn" 15% of its hydrogen. The sun initially was made up of 70% hydrogen by mass. For each 1 gram of hydrogen burned, 10^{-5} grams of energy are removed.

   (a) Using the relation $E = mc^2$, where E is the energy in Joules, m is the mass in kilograms, and $c = 3 \times 10^8$ m/s is the speed of light, how many *Joules* of energy are released when 1 *kilogram* of hydrogen is burned?

   (b) Based on this, how much energy is available from fusion?

   (c) Given the sun's luminosity (which is given in Watts $\equiv$ Joules/second), what is the lifetime of the sun?

2. Here we return to the proverbial "flies in a box" problems of dimensional analysis with length, volumes, and densities.

   (a) The average density of stars in our solar neighborhood is 0.1 pc^{-3} (0.1 stars per cubic parsec). What is the average separation between stars, in our solar neighborhood? The closest star system to ours is α Centauri, located 1.3 pc away. Is it *closer* or *farther* than the average distance?

   (b) The 50 largest metropolitan areas in the United States have populations larger than 1 million people, with a total area of 9 million km^{-2}. If these metropolises were distributed evenly across its land mass, and given what you know about areal density, what would be the average separation between 2 metropolises?