Problem Set #1

1. Shown below is a table listing the planets' sizes, distances from the sun, and orbital periods (in days or years).

planet	mass ($\times M_{\oplus}$)	diameter (km)	density (g/cm^3)	rotation	axis tilt (deg)	distance (A.U.)	orbital period
Mercury	0.0553	4880	5.43	58.81 d	0.1	0.387	87.97 d
Venus	0.815	12,104	5.2	243.69 d	177.3	0.723	224.70 d
Earth	1	12,742	5.52	23.9345 h	23.45	1	365.26 d
Mars	0.107	6780	3.93	24.623 h	25.19	1.524	686.98 d
Jupiter	317.83	139,822	1.33	9.925 h	3.12	5.203	11.86 y
Saturn	95.162	116,464	0.687	10.50 h	26.73	9.539	29.46 y
Uranus	14.536	50,724	1.32	17.24 h	97.86	19,182	84.01 y
Neptune	17.147	49,248	1.64	16.11 h	29.56	30.06	164.79 y
Pluto	0.0021	2274	2.05	6.405 d	122.46	39.53	247.68 y

   (a) Let Earth have a diameter of 1 cm, and suppose it orbits the sun in 1 second. Replace the planets' sizes, distances, and orbital periods with their corresponding proportions (for example, a planet with a diameter of 6000 km would be something with a size of about 0.5 cm).
   The trick is to do a proportionality that looks like the following. For distances this becomes:

$$L_{\text{new}} = \frac{L_{\text{old}}}{R_{\oplus}} \times 1 \text{ cm}$$

   And for times this becomes:

$$T_{\text{new}} = \frac{T_{\text{old}}}{T_{\oplus}} \times 1 \text{ second}$$

   Where $R_{\oplus}$ is the radius of the earth (12,742 km) and $T_{\oplus}$ is Earth's orbital period (1 year, or 365.26 days). This is tabulated down below:

planet	diameter (cm)	distance (m)	orbital period (s)
Mercury	0.38	45.6	0.24
Venus	0.95	85.1	0.62
Earth	1.00	117.7	1.00
Mars	0.53	179.4	1.88
Jupiter	11.0	612.5	11.86
Saturn	9.14	1123	29.46
Uranus	3.98	2258	84.01
Neptune	3.87	3539	164.79
Pluto	0.178	4654	247.68

   I will accept the right answer to the number of significant figures you choose. However, I will not accept null results (zero, for example), although it may be the right answer to 1 significant figure in the units you choose.

   (b) The sun has a radius of 7×10^5 km, and it rotates once every 30 days. What is its corresponding size and rotation period?
   See (1a) above. Diameter = 109.9 cm, radius = 54.9 cm, period = 0.082 seconds.

   (c) Recall that a light-year is the distance light travels in 1 year. The observable universe is 20 billion light years in diameter. How large is it in kilometers.
   Here was a bit of confusion. I meant using the above proportions, how large is the universe? Given that one light-year = 10^{13} km, one gets an answer of 15.7 light-years.

2. Now suppose the sun is a 100 W lightbulb. The density of stars is 1 star/$\left(5^3 \text{ km}^3\right)$. We can see a 100 Watt lightbulb out to 80 km. How many stars can we see? **Hint: the volume of a sphere of radius R is $\frac{4}{3}\pi R^3$.**
   Here, we use dimensional arguments. The number density ρ is in units of km^{-3}. I have given a volume, so the total number of stars we can see is:

$$N = \rho V = \frac{4}{3}\pi \left(80\right)^3 5^{-3} = 17,157 \text{ "stars"}$$

   Recall $\pi \approx 3.1415$.