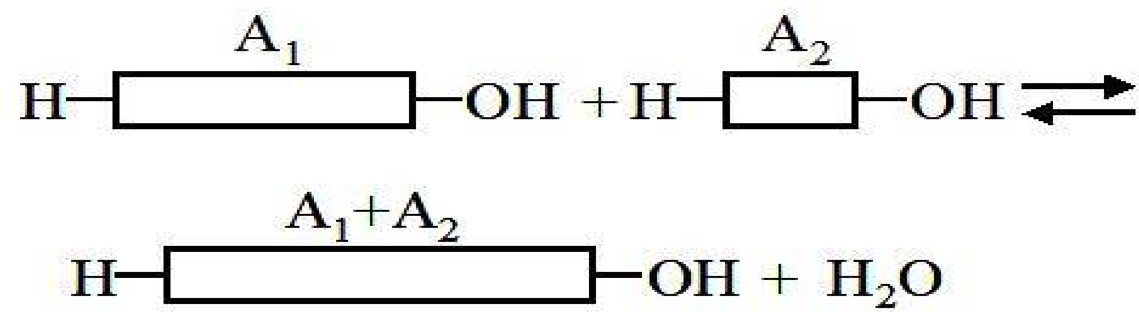
Test #3

You will have 180 minutes to complete this test. Please use your own paper in answering the questions on this test. You should use a calculator to answer the numerical questions on this test.

10 Point Essays – Choose 6 out of 8

1. Explain how the following equilibrium chemical reaction



   Tends to lead to the depolymerization of amino acids in water solutions. Recall that the chemical symbol of water is H_2O.
   Recall that for chemical equilibrium, that if we decrease the quantity of H_2O, then the reaction will go to the left and vice-versa. However, since the reactions take place in a water solution, the H_2O is overwhelmingly more concentrated, in general, than either the joined or separated amino acids. Therefore, amino acids will tend to depolymerize in water solutions unless they are highly concentrated.
2. Describe the physical process that is expected to bring "hot Jupiters" much closer to their parent stars, and the main problem associated with this physical process.
   Hot Jupiters are expected to be drawn closer to their parent stars due to friction with the gas and dust in the disk. The problem is in how to stop this frictional process, in order to keep the giant planet from crashing into the star.
3. Typical models of carbon-based life using liquid water as a solution give an upper and lower limit to the mass of stars that can establish intelligent life. Why the upper limit, and why the lower limit?
   The upper limit is placed due to the finite lifetime (4 billion years) for the development of intelligent life from nonlife. The lower limit is placed because for sufficiently low-massed stars, a planet in the star's CHZ (continuously habitable zone) would become tidally locked – rendering it uninhabitable.
4. What is the main issue with the Miller-Urey experiment that caused it to be largely invalidated – that is, what is the a ssumption made on the composition of the early atmosphere that Miller and Urey made when they performed their experiments? Why was this early atmosphere composition invalidated?
   The Miller-Urey experiment assumed the existence of a heavily reducing atmosphere (significant quantities of atmospheric H_2, methane, and ammonia) in order to show how the atmosphere + energy (lightning or UV) could rain down organic chemicals, which then form amino acids. It was found by several groups

that atmospheric methane and ammonia would be quickly destroyed through UV radiation (on the matter of a few decades).

5. Why is it unlikely that we will understand a signal inadvertently sent to us by an alien intelligence? Furthermore, why is it likely that an alien would encode its signal? Give an example of encoding of digital signals.
   Because the alien signal would be encoded and compressed, perhaps in a lossy format, in order to save information and hence bandwidth in transmitting the signal. However, for us it is already a difficult proposition in determining if a signal is due to extraterrestrial intelligence – if they have given no "obvious" way for us to decode the signal, then it would probably be impossible to decode it. An example of encoding is MPEG-3 (mp3), which is a protocol to compress sounds in a lossy manner so that OUR ears and brain will not think it is significantly different from normal sound. Another is the JPEG format, which is an algorithm to compress image files.
6. Why would a space-based civilization decide to colonize asteroids and comets rather than planets? If the Earth were divided into 10^{18} globes of equal size, by how much would the total surface area increase?
   Because the surface area is much larger for a given volume or mass of material, leading to much more efficient exploitation and habitation. Second, in space these asteroids and comets have negligible gravity, allowing for much cheaper transportation between them. An object divided into 10^{18} equal spherical shapes would each have 10^{-6} the diameter of the original earth, and 10^{-12} of the area. Therefore the total area becomes $10^{-12} \times 10^{18} = 10^{6}$ times the area of the original earth.
7. What are the benefits of using the carbonate-silicate cycle to provide for the level of atmospheric CO_2? Was the greenhouse effect larger or smaller on the primeval Earth, and why?
   The carbonate-silicate cycle has a negative feedback and is stable – that is, if the Earth gets more sunlight, then the carbonate-silicate cycle will lead to smaller atmospheric CO_2 concentrations and reducing the greenhouse effect and vice-versa. The greenhouse effect had to be larger on the primeval earth because the sun was dimmer, but the oceans were not always frozen over (they were frozen for at least two times over 1 billion years ago).
8. Give two pieces of evidence of major impact events (after the formation of the planets) in the early solar system.
   The formation of the moon, from the impact of a Mars-sized object with Earth.
   The slow rotation period of Venus.
   The fact that Uranus's axis of rotation is nearly aligned with the ecliptic.
   Pluto and Charon as orphans from one of the gas giants?
   The Caloris impact crater on Mercury.
   Asteroid belt?

30 Point Essays – Choose 4 out of 6

1. Given that the angular separation of an object (or a change in distance) of size D, as seen from a distance R, is θ=D/R. Using this result, we will answer questions about astrometry.
   a. (5 points) Assume a Jupiter-sized object, with mass $M_{Jupiter} = 2\times10^{27}$ kg orbiting at 5 AU around a 1 solar mass star, $M_{star} = 2\times10^{30}$ kg. We observe the wobble of the parent star, noting that $D_{star} = M_{star} / M_{planet} \times D_{planet}$. What is the wobble, in AU, of the star about the star-planet center of mass?
   Using the above formula, $D_{star} = 0.001\times5 = 0.005$ **AU.**
   b. (5 points) This star system is located at a distance of 25 parsecs, with 1 parsec = 3.26 light-years, and 1 light-year = 10^{16} meters. What is the angular separation of the star in its wobble, in arcseconds? 1 arcsecond = 1/3600 degrees, and π radians fill up 180 degrees.
   The distance in meters becomes $D_{star} = 0.005\times1.5\times10^{11} = 7.5\times10^{8}$ **meters. The distance to the star is** $R_{star} = 25\times3.2\times10^{16} = 8.15\times10^{17}$ **meters. Then the angle** $\theta = 7.5\times10^{8}/(8.15\times10^{17}) = 9.20\times10^{-10}$ **radians. This angle is then given by** $\theta = 9.20\times10^{-10}\times(180/\pi)\times3600 = 1.89\times10^{-4}$ **arcseconds.**
   c. (5 points) Now suppose that a spectroscopist wants to measure the velocity shift. A planet takes 11.2 years to orbit a star of the mass of the sun at a distance of 5 AU. What is the orbital velocity of the planet in its orbit around the star (technically speaking, the star-planet center of mass)?
   The planet spins out a circumference distance $C = 2\pi D_{planet} = 2\pi\times(7.5\times10^{8}) = 4.71\times10^{9}$ **kilometers. It takes** $T = 86400\times365.26\times11.2 = 3.53\times10^{8}$ **seconds to revolve around the sun. Therefore the velocity of the planet** $V_{planet} = C/T = 13.3$ **kilometers/second.**
   d. (5 points) The velocity of the star about the center of mass, what a spectroscopist would measure, is $V_{star} = M_{star} / M_{planet} \times V_{planet}$. What is the velocity of the star about the star-planet center of mass?
   Using the above formula, $V_{star} = 0.001\times13300 = 13.3$ **meters/second.**
   e. (5 points) What a spectroscopist might naively measure is the Doppler shift of an absorption line in the star's spectrum. The Doppler shift of an absorption line is $\Delta\lambda/\lambda = V_{star}/c$, where c = 3 x 10^{8} m/s, where λ is the wavelength of the absorption line and Δλ is the shift in the wavelength. What is the shift, in nanometers, associated with a 656 nanometer absorption line?
   Using the above formula, the shift $\Delta\lambda = 13.3/(3\times10^{8})656 = 2.91\times10^{-5}$ **nanometers.**
   f. (5 points) There is a characteristic width of absorption lines due to the thermal motion of hydrogen. Given that $V_{thermal} = \sqrt{3k_B T / m_{hydrogen}}$, $m_{hydrogen}$ = 1.7 x

10^{-26} kg, $k_B = 1.38 \times 10^{-23}$ Joules/Kelvin, and the atmosphere is at 6000 Kelvin, what is the width, in nanometers, of a 656 nanometer absorption line due to thermal motion of hydrogen atoms?

First calculate the thermal velocity,

$V_{thermal} = \sqrt{3(1.38\times10^{-23})(6000)/(1.7\times10^{-26})} = 3820$ **meters/second.**

Then the width of the line associated with thermal motion of the gas, $\Delta\lambda = 3820/(3\times10^{8})656 = 8.36\times10^{-3}$ **nanometers. The width of the line is much larger than the periodic shift due to Doppler motion of the star.**

2. The Bekenstein bound places a limit on the information content of a typical bit of localized information. The maximum number of bits $I_{Bekenstein} = E\,D/(h\,c)$, where E is the energy of the object, D is the size of the object, $h = 6.7 \times 10^{-34}$ Joule-seconds is Planck's constant, and $c = 3 \times 10^{8}$ m/s, and $I_{Bekenstein}$ is given in bits.
   a. (5 points) The brain has a mass of 1 kilograms, occupies a space of 10 centimeters, and has 10^{16} bits of information. How does this compare to the Bekenstein bound for something with the mass and size of the human brain? Remember $E = Mc^2$.

      The total energy available in 1 kg is $E = (3\times10^{8})^2 = 9\times10^{16}$ **Joules. The size of the brain is** $D = 0.1$ **meters. Therefore the Bekenstein bound on the information in the brain,** $I_{Bekenstein} = (9\times10^{16})(0.1)/(6.7\times10^{-34}\times3\times10^{8}) = 4.48\times10^{40}$ **bits. The human brain, with 10^{16} bits, is only** $10^{16}/(4.48\times10^{40}) = 2.23\times10^{-25}$ **of the Bekenstein bound.**
   b. (5 points) The minimum size of an object is limited by that size at which the object will collapse into a black hole, and is given by $D_{Schwarzchild} = GM/c^2$, where $G = 6.67 \times 10^{-11}$ kg^{-1} m^3 s^{-2}. For a 1 kilogram object, what is the absolute Bekenstein bound on the information content?

      Using the above relation, replacing E with Mc^2 and D with GM/c^2 yields that

      $I^{max}_{Bekenstein} = GM^2/(hc) = (6.673\times10^{-11})1^2/(6.7\times10^{-34}\times3\times10^{8}) = 3.32\times10^{14}$ **bits**

      .
   c. (20 points) The observable "universe" can be modeled as a "black hole" with size $D_{Schwarzchild}$ = 10 billion light-years. Given the calculated mass of the universe and its size, what is the maximum allowable information content in the whole universe, in bits?

      Now with the above substitutions, **$D = GM/c^2$ and $E = Mc^2$ implies that $M = Dc^2/G$ and so $E = Mc^2 = Dc^4/G$. The size of the Universe is** $D = 10^{10}\times10^{16} = 10^{26}$ **meters. The Bekenstein bound for the black hole, in terms of D becomes:**

      $I^{max}_{Bekenstein} = D^2c^3/(hG) = (10^{26})^2(3\times10^{8})^3/(6.7\times10^{-34}\times6.626\times10^{-11}) = 6.08\times10^{120}$

      bits.

3. Typical beacons used in communication might be directional rather than omnidirectional. A typical receiver has a specific gain $G > 1$, where $1/G$ is the fraction of the whole sky to which the detector is transmitting. For $G = 1$, the luminosity of radiation entering the detector is given by $L_{detect} = A_{detect} L_{source}/(4\pi R^2)$, where L_{source} is the source luminosity, A_{detect} is the detector area, and R is the separation between detector and source. For a transmitter with finite gain $G > 1$, $L_{detect} = GA_{detect}L_{source}/(4\pi R^2)$.
   a. (10 points) Suppose the omnidirectional transmitter can transmit to a standard detector, with area A_{detect} and threshold power sensitivity L_{detect}, out to a distance R_{omni}. What is the distance that a directed transmitter's signal, with gain G, can be detected with the same type of detector – that is, what is $R_{directed}/R_{omni}$ in terms of G?
   The trick here is to note that the luminosity of the transmitter, the luminosity of radiation entering the detector, and the area of the detector remain unchanged between the two cases (omnidirectional and directed). Take the following ratio:

$$\frac{L_{detect}}{L_{detect}} = \frac{\frac{L_{source}}{4\pi R_{omni}^2} A_{detect}}{\frac{GL_{source}}{4\pi R_{directed}^2} A_{detect}}$$

$$\left(\frac{R_{directed}}{R_{omni}}\right)^2 G^{-1} = 1$$

$$\frac{R_{directed}}{R_{omni}} = G^{1/2}$$

   As expected, the directed transmitter can transmit out to a distance further than the omnidirectional transmitter.
   b. (10 points) The density of stars (number/volume) in space is n. What is the total number of stars N_{omni} that a transmitter can send out to, if it can send out to a sphere of radius R_{omni}?
   From here, $N_{omni} = nV = \frac{4}{3}\pi R_{omni}^3 n$ **, since it transmits into a sphere.**
   c. (10 points) Note that a directed transmitter can send out to a distance $R_{directed}$, but covering a volume $4\pi R_{directed}^3/(3G)$. What is $N_{directed}$, the number of stars to which this transmitter's signal can be detected, in terms of N_{omni} and G – that is, what is $N_{directed}/ N_{omni}$ in terms of G?
   The number of stars reached through the directed transmitter is given by:

$$N_{directed} = nV = \frac{4}{3G}\pi R_{directed}^3 n$$

   Therefore,

$$\frac{N_{directed}}{N_{omni}} = \frac{\frac{4}{3G}\pi R^3_{directed} n}{\frac{4}{3}\pi R^3_{omni} n} = \left(\frac{R_{directed}}{R_{omni}}\right)^3 G^{-1} = G^{3/2} G^{-1} = G^{1/2}$$

Strangely enough, a directed transmitter can reach MORE stars than an omnidirectional one!

4. Now consider the stresses on typical space habitats – these are structures that need, largely, to support their own weight.
   a. (15 points) Using dimensional analysis, construct a Pressure (Force/Area, or dimensionally $M L^{-1} T^{-2}$) from the following parameters: ρ (mass volume density, units of $M L^{-3}$), a (units of acceleration, or $L T^{-2}$), and R (units of length or L).
   Using the dimensionality prescription, $P = \rho^{\alpha} a^{\beta} R^{\gamma}$ **. Then putting in the dimensional quantities:**
   $$ML^{-1}T^{-2} = \left(ML^{-3}\right)^{\alpha} \left(LT^{-2}\right)^{\beta} L^{\gamma}$$
   $$ML^{-1}T^{-2} = M^{\alpha} L^{-3\alpha+\beta+\gamma} T^{-2\beta}$$
   So that we have 3 systems of equations to solve for 3 unknowns.
   $$\alpha = 1$$
   $$-3\alpha + \beta + \gamma = -1$$
   $$-2\beta = -2$$
   Which has the solution $\alpha = 1$**,** $\beta = 1$**, and** $\gamma = 1$**. Therefore, the pressure on the supporting materials in a space habitat,** $P = \rho \times a \times R$**.**
   b. (7 points) What is the radius of a cylindrical space habitat that rotates once about its axis every 5 minutes and simulates 1 Earth gravity (10 m/s^2) at its inner edge? Recall that $a = \omega^2 R$, where $\omega = 2\pi/T$, and T being the period of rotation.
   The period of rotation is 5 minutes = 300 seconds, therefore
   $\omega = 2\pi / 300 = 2.09 \times 10^{-2}$ **radians/second. The acceleration a = 10 m/s^2. Therefore, the radius of the habitat**
   $R = a/\omega^2 = 10/\left(2.09\times10^{-2}\right)^2 = 2.28\times10^4$ **meters.**
   c. (8 points) Copper steel has a density of 6400 kg/m^3 and will break down at pressures of 3×10^8 N/m^2 (3×10^8 kg m^{-1} s^{-2}). Will this substance be an adequate material in the above space habitat?
   Using the formula derived from (a), the pressure on the structural support, if it is made of copper steel, $P = 6400 \times 10 \times 2.28\times10^4 = 1.46\times10^9$ **kg m^{-1} s^{-2}, so this substance is unsuitable for forming the structural material of the habitat.**

5. The Doomsday Argument uses, as "proof," that we will probably die within the next few centuries based on the following assumptions: 1) the Copernican principles – we are not special, and 2) we do not know the future of our species. Assume that you are equally likely to be born at any order of all the people in the human race.

a. (10 points) Given that there have been 100 billion people to have been born to the human race from the beginnings of *Homo Sapien's* existence. The 95% interval of the number of human beings being born is that this $N_{current}$, the current number of humans born, is either 2.5% or 100-2.5 = 97.5% of everyone who has been born. What is the range of N_{total}, the total number of people born to the human race?
If the total number of people born is 97.5% of the people who have been born, then $N_{total} = 10^{11}/0.975 = 1.025\times10^{11}$ people. However, if the total number of people born is 2.5% of the total, then $N_{total} = 10^{11}/0.025 = 4\times10^{12}$. Therefore, the possible number of people born into the human race becomes $1.025\times10^{11} \le N_{total} \le 4\times10^{12}$.

b. (5 points) Suppose Earth has a population that stabilizes at 10 billion people. The average life expectancy becomes 100 years. What must be the birth rate that will stabilize the human population at this level? You can use dimensional analysis to figure this out.
If there are 10 billion people on Earth, and they live on average 100 years, then the number of people who die PER YEAR is 10 billion/100 = 100 million. The population is stabilized at 10 billion people, so the number of people born must equal the number dying, therefore the birth rate is 100 million births/year.

c. (15 points) Assuming a constant birth rate given by the above, and the range of N_{total}, what is the range in the number of years for the lifetime of the human race from now?
Given the above ranges, the new number of people is $N_{new} = N_{total} - N_{current}$.
Therefore, the number of new people to be born to the human race becomes $2.56\times10^{9} \le N_{total} \le 3.9\times10^{12}$. Given that the birth rate is 10^{8} births/year, and assuming this birth rate continues, the future lifetime of the human race becomes $25.6\text{ years} \le N_{total} \le 39000\text{years}$.

6. An aggressive alien species wants to destroy the sun and mine it for energy! Is this process "economic"? Here you will answer this question.
   a. (10 points) Using $G=M^{-1}L^{3}T^{-2}$, M (the mass of the sun), and R (the sun's radius), construct something with units of energy E – that is, $E = G^{\alpha}M^{\beta}R^{\gamma}$.
   Using the dimensional prescription, $E = G^{\alpha}M^{\beta}R^{\gamma}$. Then putting in the dimensional quantities:

$$ML^{2}T^{-2} = \left(M^{-1}L^{3}T^{-2}\right)^{\alpha}\left(M\right)^{\beta}L^{\gamma}$$

$$ML^{2}T^{-2} = M^{-\alpha+\beta}L^{3\alpha+\gamma}T^{-2\alpha}$$

So that we have 3 systems of equations to solve for 3 unknowns.

$$-\alpha+\beta=1$$

$$3\alpha+\gamma=2$$

$$-2\alpha=-2$$

Which has the solution $\alpha = 1$**,** $\beta = 2$**, and** $\gamma = -1$**. So the potential energy** $E = GM^2R^{-1}$**.**

b. (5 points) Given that the mass of the sun is M = 2 x 10^{30} kg, its radius R = 6 x 10^9 m, and G = 6.673 x 10^{-11} kg^{-1} m^3 s^{-2}, what is the gravitational potential energy of the sun?

From formula calculated in (a), the potential energy of the sun (the energy required for blowing it up) is

$E_{potential} = (6.673 \times 10^{-11})(2 \times 10^{30})^2 / (6 \times 10^9) = 4.45 \times 10^{40}$ **Joules.**

c. (5 points) The total energy that can be liberated from hydrogen as it is fused into iron is 8.8 MeV/proton. The mass of the proton is 980 MeV. What is the efficiency, in the conversion of matter into energy, of fusion into iron?

Fusion can liberate 8.8 MeV per proton. The mass of a proton is 980 MeV. Therefore the efficiency of the reaction is 8.8/980 = 0.00898 = 0.898%.

d. (10 points) The sun is 70% hydrogen by mass. Therefore, how much energy (in Joules) is available from fusion? How does this compare to the sun's potential energy, hence energy required to blow up the sun?

The mass available for fusion is $M_{fusion} = 0.7 \times 2 \times 10^{30} = 1.4 \times 10^{30}$ **kg. The efficiency of a fusion is 0.898%, therefore the energy from fusion become** $M_{energy} = 1.4 \times 10^{30} \times 0.00898 = 1.26 \times 10^{28}$ **kilograms. Using the E = Mc^2 formula, the energy from fusion becomes**

$E_{fusion} = 1.26 \times 10^{28} \times (3 \times 10^8)^2 = 1.13 \times 10^{45}$ **Joules of energy. This is approximately 25,400 times the potential energy of the sun – the energy required to blow up the sun.**

Personally, I would not recommend they blow up the sun. There are plenty of other stars – such as red dwarfs – that would be more "economical" in terms of mining. Also, there are problems in that latter-stage fusion processes produce a lot of neutrinos, which we cannot harness.