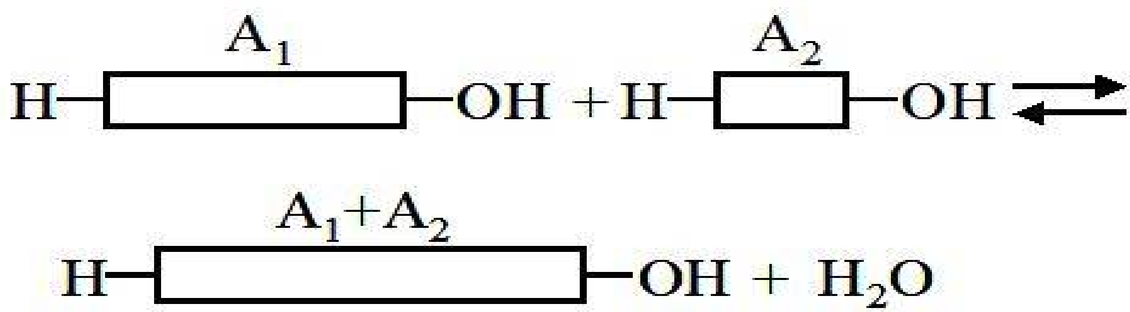
Test #3

You will have 180 minutes to complete this test. Please use your own paper in answering the questions on this test. You should use a calculator to answer the numerical questions on this test.

10 Point Essays – Choose 6 out of 8

1. Explain how the following equilibrium chemical reaction



   Tends to lead to the depolymerization of amino acids in water solutions. Recall that the chemical symbol of water is H_2O.
2. Describe the physical process that is expected to bring “hot Jupiters” much closer to their parent stars, and the main problem associated with this physical process.
3. Typical models of carbon-based life using liquid water as a solution give an upper and lower limit to the mass of stars that can establish intelligent life. Why the upper limit, and why the lower limit?
4. What is the main issue with the Miller-Urey experiment that caused it to be largely invalidated – that is, what is the assumption made on the composition of the early atmosphere that Miller and Urey made when they performed their experiments? Why was this early atmosphere composition invalidated?
5. Why is it unlikely that we will understand a signal inadvertently sent to us by an alien intelligence? Furthermore, why is it likely that an alien would encode its signal? Give an example of encoding of digital signals.
6. Why would a space-based civilization decide to colonize asteroids and comets rather than planets? If the Earth were divided into 10^{18} globes of equal size, by how much would the total surface area increase?
7. What are the benefits of using the carbonate-silicate cycle to provide for the level of atmospheric CO_2? Was the greenhouse effect larger or smaller on the primeval Earth, and why?
8. Give two pieces of evidence of major impact events (after the formation of the planets) in the early solar system.

30 Point Essays – Choose 4 out of 6

1. Given that the angular separation of an object (or a change in distance) of size D, as seen from a distance R, is θ=D/R. Using this result, we will answer questions about astrometry.
   a. (5 points) Assume a Jupiter-sized object, with mass $M_{Jupiter}=2x10^{27}$ kg orbiting at 5 AU around a 1 solar mass star, $M_{star}=2x10^{30}$ kg. We observe the wobble of the parent star, noting that $D = M_{planet}/M_{star} \times D_{orbit}$. What is the wobble, in AU, of the star about the star-planet center of mass?
   b. (5 points) This star system is located at a distance of 25 parsecs, with 1 parsec = 3.26 light-years, and 1 light-year = 10^{16} meters. What is the angular separation of the star in its wobble, in arcseconds? 1 arcsecond = 1/3600 degrees, and π radians fill up 180 degrees.
   c. (5 points) Now suppose that a spectroscopist wants to measure the velocity shift. A planet takes 11.2 years to orbit a star of the mass of the sun at a distance of 5 AU. What is the orbital velocity of the planet in its orbit around the star (technically speaking, the star-planet center of mass)?
   d. (5 points) The velocity of the star about the center of mass, what a spectroscopist would measure, is $V_{star} = M_{planet}/M_{star} \times V_{planet}$. What is the velocity of the star about the star-planet center of mass?
   e. (5 points) What a spectroscopist might naively measure is the Doppler shift of an absorption line in the star's spectrum. The Doppler shift of an absorption line is $\Delta\lambda/\lambda = V_{star}/c$, where $c = 3 \times 10^8$ m/s, where λ is the wavelength of the absorption line and $\Delta\lambda$ is the shift in the wavelength. What is the shift, in nanometers, associated with a 656 nanometer absorption line?
   f. (5 points) There is a characteristic width of absorption lines due to the thermal motion of hydrogen. Given that $V_{thermal} = (3k_BT/m_{hydrogen})^{1/2}$, $m_{hydrogen} = 1.7 \times 10^{-26}$ kg, $k_B = 1.38 \times 10^{-23}$ Joules/Kelvin, and the atmosphere is at 6000 Kelvin, what is the width, in nanometers, of a 656 nanometer absorption line due to thermal motion of hydrogen atoms?

2. The Bekenstein bound places a limit on the information content of a typical bit of localized information. The maximum number of bits $I_{Bekenstein} = E\,D/(h\,c)$, where E is the energy of the object, D is the size of the object, $h = 6.7 \times 10^{-34}$ Joule-seconds is Planck's constant, and $c = 3 \times 10^8$ m/s, and $I_{Bekenstein}$ is given in bits.
   a. (5 points) The brain has a mass of 1 kilograms, occupies a space of 10 centimeters, and has 10^{16} bits of information. How does this compare to the Bekenstein bound for something with the mass and size of the human brain? Remember $E = Mc^2$.
   b. (5 points) The minimum size of an object is limited by that size at which the object will collapse into a black hole, and is given by $D_{Schwarzchild} = GM/c^2$, where $G = 6.67 \times 10^{-11}$ kg^{-1} m^3 s^{-2}. For a 1 kilogram object, what is the absolute Bekenstein bound on the information content?
   c. (20 points) The observable "universe" can be modeled as a "black hole" with size $D_{Schwarzchild}$ = 10 billion light-years. Given the calculated mass of the

universe and its size, what is the maximum allowable information content in the whole universe, in bits?

3. Typical beacons used in communication might be directional rather than omnidirectional. A typical receiver has a specific gain $G > 1$, where $1/G$ is the fraction of the whole sky to which the detector is transmitting. For $G = 1$, the luminosity of radiation entering the detector is given by $L_{detect} = A_{detect} L_{source}/(4\pi R^2)$, where L_{source} is the source luminosity, A_{detect} is the detector area, and R is the separation between detector and source. For a transmitter with finite gain $G > 1$, $L_{detect} = GA_{detect}L_{source}/(4\pi R^2)$.
    a. (10 points) Suppose the omnidirectional transmitter can transmit to a standard detector, with area A_{detect} and threshold power sensitivity L_{detect}, out to a distance R_{omni}. What is the distance that a directed transmitter's signal, with gain G, can be detected with the same type of detector – that is, what is $R_{directed}/R_{omni}$ in terms of G?
    b. (10 points) The density of stars (number/volume) in space is n. What is the total number of stars N_{omni} that a transmitter can send out to, if it can send out to a sphere of radius R_{omni}?
    c. (10 points) Note that a directed transmitter can send out to a distance $R_{directed}$, but covering a volume $4\pi R_{directed}^3/(3G)$. What is $N_{directed}$, the number of stars to which this transmitter's signal can be detected, in terms of N_{omni} and G – that is, what is $N_{directed}/ N_{omni}$ in terms of G?

4. Now consider the stresses on typical space habitats – these are structures that need, largely, to support their own weight.
    a. (15 points) Using dimensional analysis, construct a Pressure (Force/Area, or dimensionally $M L^{-1} T^{-2}$) from the following parameters: ρ (mass volume density, units of $M L^{-3}$), a (units of acceleration, or $L T^{-2}$), and R (units of length or L).
    b. (7 points) What is the radius of a cylindrical space habitat that rotates once about its axis every 5 minutes and simulates 1 Earth gravity (10 m/s^2) at its inner edge? Recall that $a = \omega^2 R$, where $\omega = 2\pi/T$, and T being the period of rotation.
    c. (8 points) Copper steel has a density of 6400 kg/m^3 and will break down at pressures of 3×10^8 N/m^2 (3×10^8 kg m^{-1} s^{-2}). Will this substance be an adequate material in the above space habitat?

5. The Doomsday Argument uses, as "proof," that we will probably die within the next few centuries based on the following assumptions: 1) the Copernican principles – we are not special, and 2) we do not know the future of our species. Assume that you are equally likely to be born at any order of all the people in the human race.
    a. (10 points) Given that there have been 100 billion people to have been born to the human race from the beginnings of *Homo Sapien's* existence. The 95% interval of the number of human beings being born is that this $N_{current}$, the current number of humans born, is either 2.5% or 100-2.5 = 97.5% of

everyone who has been born. What is the range of N_{total}, the total number of people born to the human race?

b. (5 points) Suppose Earth has a population that stabilizes at 10 billion people. The average life expectancy becomes 100 years. What must be the birth rate that will stabilize the human population at this level? You can use dimensional analysis to figure this out.

c. (15 points) Assuming a constant birth rate given by the above, and the range of N_{total}, what is the range in the number of years for the lifetime of the human race from now?

6. An aggressive alien species wants to destroy the sun and mine it for energy! Is this process "economic"? Here you will answer this question.
   a. (10 points) Using $G=M^{-1}L^{3}T^{-2}$, M (the mass of the sun), and R (the sun's radius), construct something with units of energy E – that is, $E = G^{\alpha}M^{\beta}R^{\gamma}$.
   b. (5 points) Given that the mass of the sun is $M = 2 \times 10^{30}$ kg, its radius $R = 6 \times 10^{6}$ m, and $G = 6.673 \times 10^{-11}\ kg^{-1}\ m^{3}\ s^{-2}$, what is the gravitational potential energy of the sun?
   c. (5 points) The total energy that can be liberated from hydrogen as it is fused into iron is 8.8 MeV/proton. The mass of the proton is 980 MeV. What is the efficiency, in the conversion of matter into energy, of fusion into iron?
   d. (10 points) The sun is 70% hydrogen by mass. Therefore, how much energy (in Joules) is available for turning into energy? How does this compare to the sun's potential energy, hence energy required to blow up the sun?

Personally, I would not recommend they blow up the sun. There are plenty of other stars – such as red dwarfs – that would be more "economical" in terms of mining. Also, there are problems in that latter-stage fusion processes produce a lot of neutrinos, which we cannot harness.