Review For Test #2: Answers

The material on this test will be a comprehensive and representative sampling of material to review on the first test, to be given on Monday, February 23. As such, there will be seven main topics (Extragalactic Distance Scale, Galaxy Clustering, "Dark Matter", Active Galaxies and Active Galactic Nuclei (AGN), and Cosmology). There will be four multiple choice and one essay question per test. Answers are denoted in red.

Extragalactic Distance Scale

1. Parallax is not useful as a standard candle (measurement out to megaparsecs) because:
   (a) We can see objects only out to a few kiloparsecs.
   (b) Objects in the sky move too quickly for parallax measurements to be reliable.
   (c) The angular deviation of celestial objects as the earth moves in its orbit would be too small to resolve.
   (d) Stars are much too faint to be seen outside our own galaxy.
2. The cosmic microwave background radiation:
   (a) Is 30 K thermal emission redshifted from the era of recombination.
   (b) Is isotropic to a very high level as observed by telescopes on Earth.
   (c) Is an intermittent astrophysical phenomenon.
   (d) Is 3 K microwave emission redshifted from the era of recombination.
3. Hubble's constant is 71 km s^{-1} Mpc^{-1}. An object has a redshift z = 0.001. It is located:
   (a) 43 MPc away.
   (b) 4.3 Mpc away.
   (c) 0.43 Mpc away.
   (d) 0.043 Mpc away.
4. The Sunyaev-Zeldovich effect affects the cosmic microwave background (CMB) by:
   (a) Scattering and reddening light by the dust within clusters.
   (b) slightly redshifting CMB radiation as it passes within a cluster's gravitational well.
   (c) absorption and scattering of CMB radiation by hot electrons within a cluster.
   (d) Redshifting light from the CMB as the universe expands.

Standard Distance. At maximum light, Type Ia supernovae have absolute magnitudes of $M_B = -19.6$. Assume a Type Ia supernova occurs in an extremely distant galaxy 5000 Mpc away. Estimate the observed magnitude B of the supernova at maximum light, ignoring the K-correction (effect of redshift on the spectrum), any absorption, or any cosmological effects. Could this supernova be observed with the Hubble

Space Telescope (limited to $m_B = 29$)?

The distance modulus formula gives the luminosity as:

$$m_B = M_B + 5\log_{10}(D/10 \text{ pc}) = -19.6 + 5\log_{10}\left(\frac{5\times 10^9}{10}\right) = 23.9$$

This supernova can be detected by the Hubble.

Galaxy Clustering

1. A collection of 1000 galaxies has an average velocity dispersion $v = 5000$ km/s. The cluster is 1 Mpc in radius. Its mass is:

   (a) $6 \times 10^{15}\ M_{\odot}$.

   (b) $3 \times 10^{14}\ M_{\odot}$.

   (c) $6 \times 10^{17}\ M_{\odot}$.

   (d) $6 \times 10^{14}\ M_{\odot}$.

2. Based on a velocity dispersion $v = 1000$ km/s, and the fact that the mass of a proton is $m_p = 1.7 \times 10^{-24}$ gm, and Boltzmann's constant is 1.4×10^{-16} erg/K, the temperature within a galaxy cluster is approximately:

   (a) $T = 10^7$ K.

   (b) $T = 3 \times 10^9$ K.

   (c) $T = 10^8$ K.

   (d) $T = 3 \times 10^6$ K.

3. The following mechanism is believed to describe the formation of visible structure (galaxies, clusters, perhaps superclusters) in the universe.

   (a) top-down (large → small) formation from matter bubbles blown out by hot dark matter.

   (b) Cold dark matter (of unknown composition) seeding small nuclei of matter, growing larger as time goes on.

   (c) Hot dark matter growing baryonic concentrations from small to large.

   (d) An initial seed of matter collected around cosmic strings and magnetic monopoles.

4. The two-point correlation function, which formally describes the probability of finding another galaxy within a distance r from the first galaxy, implies that:

   (a) The structure of the universe is scale-free; there is no large or small length scale for galaxy clustering.

   (b) The size of clusters has an upper limit – a length scale associated with it.

   (c) Implies a universe filled with small voids and dense clusters.

   (d) There is zero chance of finding two galaxies separated by a distance smaller than $r_{\min}$.

Cluster Virialization. Consider a cluster of 1000 galaxies, moving at average velocities of 1000 km/s, spread out over 1 Mpc. The radius of each galaxy can be taken as 50 kpc. How long does it take for galaxy-galaxy collisions in this cluster? To answer this question, answer the following:

1. Assume the cluster is a sphere of radius 1 Mpc. What is the volume occupied by each galaxy, on average?
   The volume occupied by each galaxy $V_{\text{gall}} = V/N$.

$$V_{\text{gal}} = \frac{\frac{4}{3}\pi R_{cl}^3}{N} = 4.2 \times 10^{15}\ \text{pc}^3 = 4.2 \times 10^{-3}\ \text{Mpc}^3$$

2. Each galaxy sweeps out a cylinder of radius $R = 50$ kpc and length $d = vt$ in time t. In 10^7 years, what volume, in Mpc^3, is swept out?
   The volume swept out is a cylinder with radius $r = 50$ kpc and length $d = vt$. Therefore the volume swept out by a galaxy in time t:

$$V_{\text{sweep}} = \pi r^2 vt$$

   In $t = 10^7$ years, the volume swept out is:

$$V_{\text{sweep}} = \pi \left(5 \times 10^{-2}\ \text{Mpc}\right)^2 \left(\frac{10^6 \times 86400 \times 365.25 \times 10^7}{3.26 \times 10^6 \times 86400 \times 365.25 \times 3 \times 10^8}\right)^2 = 8.0 \times 10^{-5}\ \text{Mpc}^3$$

3. The time between collisions is calculated by determining that time t during which the volume swept out by an average galaxy is the volume occupied by a given galaxy. What is this time, in Myr?
   A collision occurs, on average, when the volume swept out by a galaxy in its motion equals the volume occupied by a galaxy. Therefore we solve the following equation:

$$t = \frac{\frac{4}{3}\pi R^3}{\pi r^2 v N} = 5.2 \times 10^8\ \text{yr}$$

4. How does this compare to the age of the universe, $t_{\text{universe}} \sim 10^{10}$ years? Based on this crude argument, could galaxy-galaxy collisions equilibrate the galactic population within a cluster?
   This is much (factor of 20) smaller than the age of the universe.

"Dark Matter"

1. Most of the dark matter in the universe is not baryonic because.
   (a) It does not shine on its own through fusion reactions.
   (b) There is a maximum mass that a given collection of baryonic matter can reach at a given location in space.

(c) Baryonic matter would have scattered light too easily, making the universe opaque rather than transparent to radiation.

(d) From nucleogenetic arguments, the ratio of light isotopes implies that only a small fraction of the universe's mass is baryonic.

2. The following is known to be a true statement about dark matter.

   (a) It interacts with normal matter only through gravitational attraction.

   (b) It is dominated by fast, light, but abundant neutrinos.

   (c) Black holes are believed to be a dominant source of dark matter mass.

   (d) It is dominated by WIMPS (weakly interacting massive particles)

3. The two dominant components to the energy density of the universe are:

   (a) 1^{st}: dark matter; 2^{nd}: positive intrinsic energy density of the vacuum.

   (b) 1^{st}: positive intrinsic energy density of the vacuum; 2^{nd}: dark matter.

   (c) 1^{st}: negative intrinsic energy density of the vacuum; 2^{nd}: dark matter.

   (d) 1^{st}: photons; 2^{nd} dark matter.

4. For the given Hubble constant, the mass density of the universe due to dark matter is approximately 10^{-24} gm cm^{-3}. Assume this matter density is dominated by quantum black holes, with mass of 10^{20} gm. These black holes are separated by approximately:

   (a) 1 pc.

   (b) 10 pc.

   (c) 1 Mpc.

   (d) 0.02 pc.

Active Galaxies and Active Galactic Nuclei (AGN)

1. The broad-line emission in some Seyfert galaxies comes from the inner regions of the central engine. Some of the evidence for this is that:

   (a) Scattering by the dense plasma in the central region is large enough to broaden narrow emission lines.

   (b) Emission lines from near the central engine are due to atomic transitions that are naturally broad.

   (c) They are due to the higher temperature alone.

   (d) They are due to the fast rotational velocities close to the central black hole.

2. The radio emission from the lobes and jets of radio galaxies is due to the:

   (a) frictional interaction of the jet plasma with the surrounding medium.

(b) Synchrotron emission due to electrons gyrating in a magnetic field.
(c) Scattering of electrons in the plasma with the cosmic microwave background radiation.
(d) Synchrotron emission due to protons gyrating in a magnetic field.

3. The efficiency of radiated luminosity falling into a black hole is most comparable to:
   (a) The efficiency of fission reactions.
   (b) The efficiency of fusion reactions.
   (c) The efficiency of matter to energy conversion.
   (d) The efficiency of high-explosive chemical reactions (such as TNT).

4. A black hole of 10^7 solar masses powers a quasar, and it accretes matter at a rate of 10^{-6} $M_\odot$ yr^{-1}. Its luminosity is:
   (a) 3×10^{40} erg s^{-1}.
   (b) 3×10^{35} erg s^{-1}.
   (c) 3×10^{44} erg s^{-1}.
   (d) 3×10^{33} erg s^{-1}.

Maximum Accretion Rate Onto a Black Hole. Here we will use the Eddington limit to determine the maximum mass accretion rate $\dot{M}$ onto a black hole of mass M.

1. An electron has cross section to absorb photons of $\sigma = 7 \times 10^{-25}$ cm^{-2}. Using the radiative flux F, which has units of luminosity/area, c, the speed of light, and σ, the cross-sectional area for the absorption of photons, calculate out the *force* acting on an electron due to radiation.
   The dimensions of energy flux are luminosity/area, or energy/time/length^2. Dividing by c gets us a radiation pressure in units of energy/length^3. Multiplying by an area gets us force, therefore:
   $$F = \frac{\sigma \mathcal{F}}{c}$$
   Where F is force, σ is the absorption cross section, and $\mathcal{F}$ is the radiative flux.

2. This radiation force is balanced out by the gravitational force. Take electron to be at the Schwarzchild radius of this black hole. Derive an equation for the radiative flux that will "balance" out the gravitational force. The mass of the electron is m_e.
   The force due to radiation pressure is balanced by the gravitational acceleration at the Schwarzchild radius. The radius is given by:
   $$R_{\text{schwarzchild}} = \frac{2GM}{c^2}$$
   The gravitational acceleration at the Schwarzchild radius must balance out the radiation pressure, therefore:
   $$\frac{GMm_e}{R^2_{\text{schwarzchild}}} = \frac{c^4 m_e}{4GM} = \frac{\sigma \mathcal{F}}{c}$$
   $$\mathcal{F} = \frac{c^5 m_e}{4GM\sigma}$$

3. Derive an expression for the flux F from the mass accretion rate $\dot{M}$ and the mass of the black hole M. Assume the radiation is purely isotropic (hint, the flux $\mathcal{F} = L/\left(4\pi R_{\text{schwarzchild}}^2\right)$).
   The luminosity of a black hole is given by:

$$L = \frac{GM\dot{M}}{2R_{\text{schwarzchild}}} = \frac{1}{4}\dot{M}c^2$$

   Then the radiative flux is given by:

$$\mathcal{F} = \frac{L}{4\pi R_{\text{schwarzchild}}^2} = \frac{\dot{M}c^6}{16\pi G^2 M^2}$$

4. Equate the above and get a relation for the maximum accretion rate $\dot{M}$ for a black hole of mass M.
   Equating the fluxes derived from force balance and the flux from mass accretion:

$$\frac{\dot{M}c^6}{16\pi G^2 M^2} = \frac{m_e c^5}{4GM\sigma}$$
$$\dot{M} = \frac{4\pi G M m_e}{c\sigma} = 1.1 \times 10^{-12} \left(\frac{M}{1\,M_\odot}\right) M_\odot \text{ yr}^{-1}$$

   Any mass accretion above this limit would be throttled back to around this level.

Cosmology

1. The era of recombination occurred at a time when the energy density of the universe:
   (a) Was dominated by dark matter.
   (b) Was dominated by dark energy.
   (c) Was dominated by baryonic matter.
   (d) Was dominated by radiation.

2. A hyperbolic geometry of spacetime, in the absence of dark energy:
   (a) Is characterized by the fact that parallel lines always converge.
   (b) The volume enclosed within a sphere of radius R is larger than $\frac{4}{3}\pi R^3$.
   (c) The volume enclosed within a sphere of radius R is smaller than $\frac{4}{3}\pi R^3$.
   (d) An observational horizon that decreases in absolute size beyond a certain time.

3. The simplest models of the "quintessence" is one in which:
   (a) It has negative energy density and arbitrary (positive or negative) pressure.
   (b) It is characterized by pressure $P = -\rho_\Lambda c^2$.
   (c) It can result in the "ripping" of spacetime at some moment in the future (asymptotic runaway expansion).

(d) It can have either positive or negative energy density.

4. One of many interesting conclusions of the WMAP survey of the cosmic microwave background was that:

   (a) The size of the observational horizon is consistent with what we know from previous, independent observations.

   (b) The flatness of spacetime was confirmed to a high degree of accuracy.

   (c) Fluctuations in the background are due to overdensities in the opaque plasma, due to the interactions of baryonic with hot dark matter.

   (d) The map shows strong evidence for anomalous artifacts (cosmic strings and other topological defects in spacetime) that inflationary expansion should have diluted.

The Expansion of the Open Universe. Consider an open universe consisting of normal matter and radiation, such that $\Omega_0 < 1$. The evolution of this universe is characterized by the following equations, where R is the scale factor of the universe ($R = 1$ at the current epoch) and t is the time from the Big Bang.

$$R = \frac{\Omega_0/2}{1-\Omega_0}\left[\cosh\theta - 1\right]$$
$$H_0 t = \frac{\Omega_0/2}{\left(1-\Omega_0\right)^{3/2}}\left(\sinh\theta - \theta\right)$$

1. Show that at very late times, $\theta \to \infty$, the expansion of the universe approaches a constant value – that is, show that $dR/dt \to$ constant at late times. You may use the result that $\tanh\theta = \sinh\theta/\cosh\theta = 1$. What is this value in terms of H_0?.
   The Hubble expansion rate can be parametrized in the following manner:

$$H = \frac{dR}{dt} = \frac{dR/d\theta}{dt/d\theta}$$

   The derivatives of scaling factor and time with respect to θ:

$$\frac{dR}{d\theta} = \frac{\Omega_0/2}{1-\Omega_0}\sinh\theta$$
$$\frac{dt}{d\theta} = \frac{\Omega_0/2}{H_0\left(1-\Omega_0\right)^{3/2}}\left(\cosh\theta - 1\right)$$

   So the Hubble expansion rate:

$$H = H_0\sqrt{1-\Omega_0}\frac{\sinh\theta}{\cosh\theta - 1}$$

   As $t \to \infty$, $\theta \to \infty$ and $H \to H_0\sqrt{1-\Omega_0}\tanh\theta = H_0\sqrt{1-\Omega_0}$.

2. Set $R = 1$ and solve for θ in the above expressions, or solve for $\cosh\theta$ for the above expression.
   The parametric equation becomes:

$$R = 1 = \frac{\Omega_0/2}{1-\Omega_0}\left(\cosh\theta_0 - 1\right)$$
$$\cosh\theta_0 = 1 + \frac{1-\Omega_0}{\Omega_0/2} = \frac{2}{\Omega} - 1$$

3. Now calculate the current Hubble expansion H in terms of H_0. What value do you find?
   Here we can use the Hubble expansion formula in terms of θ:

$$H = H_0\sqrt{1-\Omega_0}\frac{\sinh\theta_0}{\cosh\theta_0 - 1}$$

   Here use the trigonometric identity, $\cosh^2\theta - \sinh^2\theta = 1$. Therefore we have that:

$$\sinh\theta_0 = \sqrt{\left(\frac{2}{\Omega_0} - 1\right)^2 - 1} = \sqrt{\frac{4}{\Omega_0}\left(\frac{1}{\Omega_0} + 1\right)}$$

   So that the current Hubble expansion rate:

$$H = H_0\sqrt{1-\Omega_0}\frac{\sqrt{\frac{4}{\Omega_0}\left(\frac{1}{\Omega_0}+1\right)}}{2\left(1-\Omega_0\right)/\Omega_0} = H_0\sqrt{\frac{1+\Omega_0}{1-\Omega_0}}$$