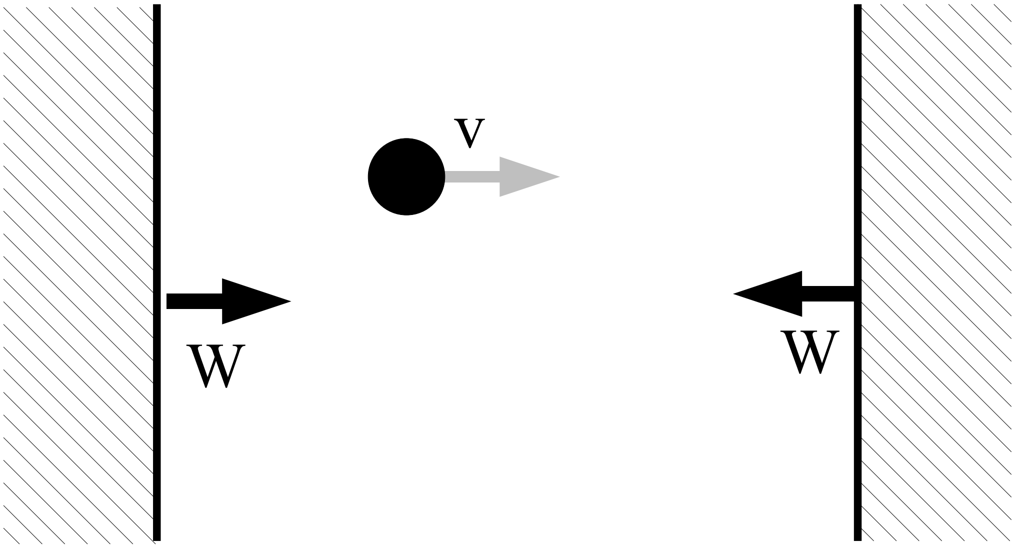
1. Let us model the interaction of a molecule hitting the side of a container as an eleastic ball hitting a wall. Imagine a ball moving between two converging walls an opposite sides. The velocity of each wall is W.



1a.) Show that each time the ball collides with either wall, its rebounds with its velocity increased by $2W$. In other words, if v is the ball velocity going into the wall, then $v + 2W$ is the ball velocity on the rebound.

1b.) Let $X(t)$ be the separation of the two walls at time t. Show that $X(t) = X_0 - At$, where A is a constant. What is the value of A?

1c.) Show that the velocity of the ball obeys the equation of motion

$$\frac{dv}{dt} = -\frac{v}{X}\frac{dX}{dt}.$$

(You may assume that X is very nearly constant for one flight time between walls, X/v.) Prove, therefore, that $vX =$ a constant.

1d.) In the a three-dimensional cube, imagine balls bouncing off all three pairs of opposite sides, one-third in the X direction, one-third in Y, and one-third in Z. The balls don't bump into each other. All three sides have the same length at all times. Using the result that the velocity of a ball times the length of one side of the cube is constant, show that the kinetic energy density of balls $\mathcal{E}$ statisfies $\mathcal{E}V^{5/3}$ is constant. Since energy density and pressure are proportional, this implies $PV^{5/3}$ is constant, where P is the pressure exerted by the balls against the wall. Note that this is the adiabatic gas condition. Evidently collisions between "ball molecules" don't matter, at least on average. As an optional exercise, you might want to think about why that is.

2a.) Find your copy of the Kepler handout. Now instead of using the equations of gravity, start with the equations of Coulomb's law:

$$M_1\frac{d^2\mathbf{r_1}}{dt^2} = \frac{Z_1Z_2e^2}{r^3}\mathbf{r}$$

$$M_2\frac{d^2\mathbf{r_2}}{dt^2} = -\frac{Z_1Z_2e^2}{r^3}\mathbf{r}$$

Here, $\mathbf{r}$ is the relative separation vector, $Z_1 e$ is the charge on particle 1 (the same for particle 2) and e is the absolute value of the electron charge. What is the energy equation (the final equation in the handout) for the Coulomb interaction? You may use the notation

$$\frac{1}{\mu} \equiv \frac{1}{M_1} + \frac{1}{M_2}$$

μ is called the "reduced mass."

2b. Two protons each have kinetic energy $1.5kT$. Taking $T = 1.5 \times 10^7$K, as in the core of the sun, what is the closest possible separation they can achieve? How does this compare with 10^{-13} cm, the maximum separation that will allow nuclear reactions to occur?

3a. The total thermal energy in the sun is 6.6×10^{48} ergs. What is the total energy (thermal plus potential) of the sun? Express your answer as a number of ergs, and as a number times $GM_{\odot}^2/R_{\odot}$.

3b. Assume that the number you just derived is always the same, even if the sun shrinks. Suppose that the sun was giving off energy by Kelvin-Helmholtz contraction, and that its mass remained constant. How fast would the radius be shrinking to power the observed luminosity? (Hint: $L = -dE(\text{total})/dt$.) Express your answer in cm per sec.

4. Shu, problem 5.9.