ASTRONOMY 211 S. Balbus
Problem Set#1 Assigned: 6 September 2004 Due: 13 September 2004

1.) The unit vectors in the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ directions are of course $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$. A particular coordinate of a vector $\mathbf{A}$ is given by, e.g., $A_x = \mathbf{A}\cdot\hat{\mathbf{x}}$, and the same for the other directions.

Suppose that we had some other set of axes pointing in three arbitrary but mutually perpendicular directions, $\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{w}}$. Then the coordinates of the vector $\mathbf{A}$ relative to these axes would just be $A_u = \mathbf{A}\cdot\hat{\mathbf{u}}$, $A_v = \mathbf{A}\cdot\hat{\mathbf{v}}$, and $A_w = \mathbf{A}\cdot\hat{\mathbf{w}}$. A very simple but important example of this is a coordinate system where the u and v axes are rotated by an angle θ relative to x and y. We'll take the w and z axes to be the same.

a.) Show, once again, that if an object has coordinates (A_x, A_y) in the original system, then in the rotated system its coordinates are:

$$A_u = A_x\cos\theta + A_y\sin\theta$$

$$A_v = -A_x\sin\theta + A_y\cos\theta$$

Yes, I know we did this in class, but I would like to see you derive this very clearly on your own. You must use a vector approach.

b.) Show that $A_x^2 + A_y^2 = A_u^2 + A_v^2$.

c.) Show explicity that if we apply the rotation law twice, i.e., we rotate the coordinates by θ and then rotate again by θ, that the resulting coordindate transformation is equal to one rotation by 2θ relative to the original x, y axes.

2a.) If two points on a sphere have the same longitude, the angle between them is just the difference in their latitudes. If two points have the same latitude, why isn't the angle between them just the difference in their longitudes?

2b.) What is the great circle distance from Peoria, Illinois to Tahiti? (Part of this problem is to find the coordinates of Peoria and Tahiti on your own, so cite your source!) The radius of the earth is 3960 miles.

3a.) At latitude l_1, the sun is directly overhead. How far above the horizon is the sun at latitude l_2? What happens when $|l_1 - l_2| > 90°$? On what day of the year does the sun drop out of the sky at the north pole? When does it appear again? How high is the sun in Charlottesville at noon on the summer solstice? The equinoxes? What is the smallest declination δ that is visible from Charlottesville? The Magellanic Clouds are at $\delta = -70°$. When, if ever, can we see them at midnight from the steps of the rotunda?

3b.) Assuming that the best time to observe a star is when it is highest overhead near midnight, which month would be best to observe the following stars: Sirius ($RA = 6^h\,43^m$), Barnard's Star ($17^h\,55^m$), Spica ($13^h\,20^m$)?

4.) The south sea island of Tongo is located at latitude 40° S. The islanders witness a conjunction of the moon with the planet Venus in the evening sky. They are so taken with this majestic sight that they make their national flag depict the event. Assuming that the flag is astronomically accurate, which of the following is a possible representation of the Tongo flag? Explain your answer convincingly!

5.) On the evening of the northern summer solstice, how long does night last in the following cities: Buenos Aires (35° S), Helsinki (60° N) London (52° N), Nairobi (1° S)?