5 6 7 8 9 10 10.95 12 14.4 17.28 20.74 24.88

Practice Problems #2, ASTR 211, Fall 2004

Answers are denoted in red.

1. The earth is aligned at an angle of 23ﬃ relative to the normal to the ecliptic. The period of Earth's precession is 26,000 years. Assume the North Star is at the North celestial pole ($\delta = 90$ﬃ) at the current epoch. What is the declination of Polaris relative to the old North celestial pole 6,500 years ago? A diagram describing this problem is shown below:

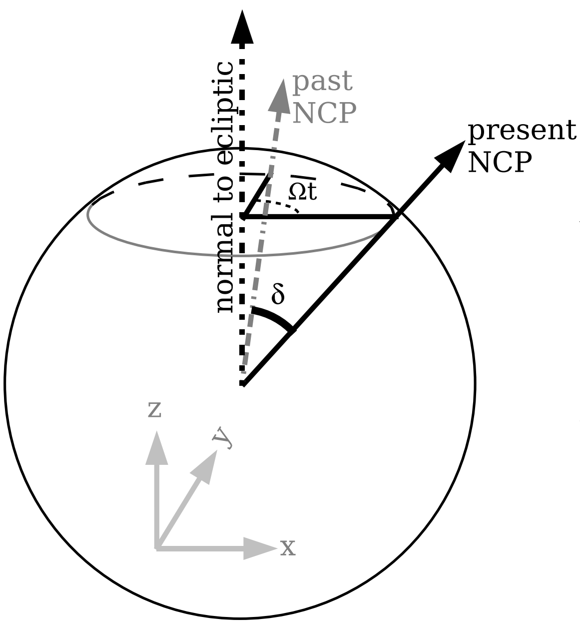

Figure 1: Diagram depicting the vector pointing at the NCP at the current epoch, and at an epoch $t = 6500$ years ago. The angle δ is the angle between the older NCP and the current NCP, or equivalently the angle between the current pole star (assumed to lie at the current NCP) and the old NCP. It is assumed that the heavens are "fixed" relative to the precession of Earth's rotation axis. Axes fixed with respect to the heavens are also shown.

At current coordinates, the unit vector $\mathbf{n}_{\rm NCP}$ has components fixed relative to the stars given by:

$$\mathbf{n}_{\rm NCP} = (\sin\chi, 0, \cos\chi)$$

Where $\chi = 23$ﬃ is the inclination of the celestial pole relative to the normal to the ecliptic. However, the precession of the north celestial pole implies that:

$$\mathbf{n}(t) = (\sin\chi\cos\Omega t, \sin\chi\sin\Omega t, \cos\chi)$$

The period of a precession is $P = 26,000$ yrs. Therefore the frequency of the precession $\Omega = 2\pi/P$. I have given the time $t = 6500$ years, allowing one to calculate out Ωt. The angle between the two vectors δ is given by:

$$\cos\delta = \mathbf{n}(t)\cdot\mathbf{n}_{\rm NCP} = \cos^2\chi + \sin^2\chi\cos\Omega t$$

The declination is the angle $\ell = 90 - \delta$.

2. Barnard's Star has some of the largest proper motions of any stellar object in the sky. Its radial velocity $v_r = 140$ km s^{-1} towards Earth, while its proper motion across the sky is $10.3''$ (10.3 arcseconds) yr^{-1}. It is located 6.7 light-years from the sun.

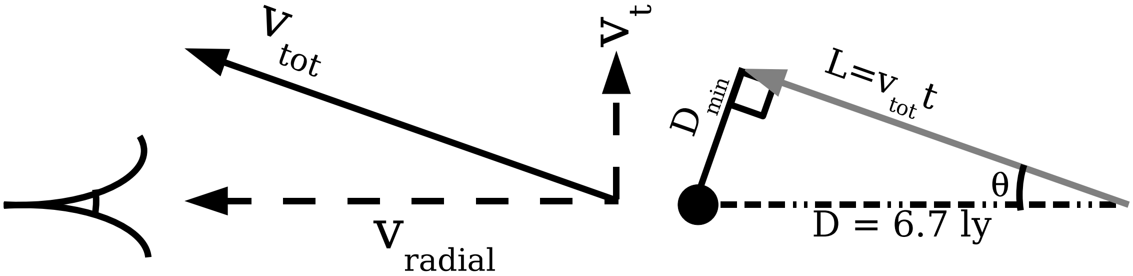

Figure 2: Diagram depicting the velocity and motion (position) of Barnard's star through time.

(a) What is its transverse velocity v_t? What is the total speed $v_{\rm tot}$ of Barnard's star? In one year, Barnard's star moves 10.3". However, the star is located 6.7 ly = 2.1 pc away. The transverse speed $v_t = 10.3 * 2.1 = 21$ AU yr^{-1} $= 1.0 \times 10^7$ cm s^{-1}.

(b) What is the angle θ that Barnard's velocity vector $\mathbf{v}_{\rm tot}$ make with our line of sight? The angle θ is defined trigonometrically to be $\tan\theta = v_t/v_r = 0.719$, therefore $\theta = 0.62$ radians $= 36$ffi.

(c) From the above diagram, what is the closest distance $D_{\rm min}$ that Barnard's star will approach the earth? How many years in the future t will this approach occur? From the above diagram, $D_{\rm min} = D\sin\theta = 3.9$ light-years. since $L = D\cos\theta$, this will occur at a time:

$$t = \frac{D\cos\theta}{\sqrt{v_r^2 + v_t^2}} = \frac{Dv_r}{v_r^2 + v_t^2} = 3.0 \times 10^{11}\ {\rm s} = 9.5 \times 10^3\ {\rm yr}$$

Therefore, around AD 11,500.

3. The Greeks observed that the time t_1 from new moon to first quarter is 15 minutes shorter than t_2, the time from first quarter to full moon. Assume the moon has a circular orbit with radius 3×10^5 km and it takes $T = 28$ days to undergo one revolution around Earth.

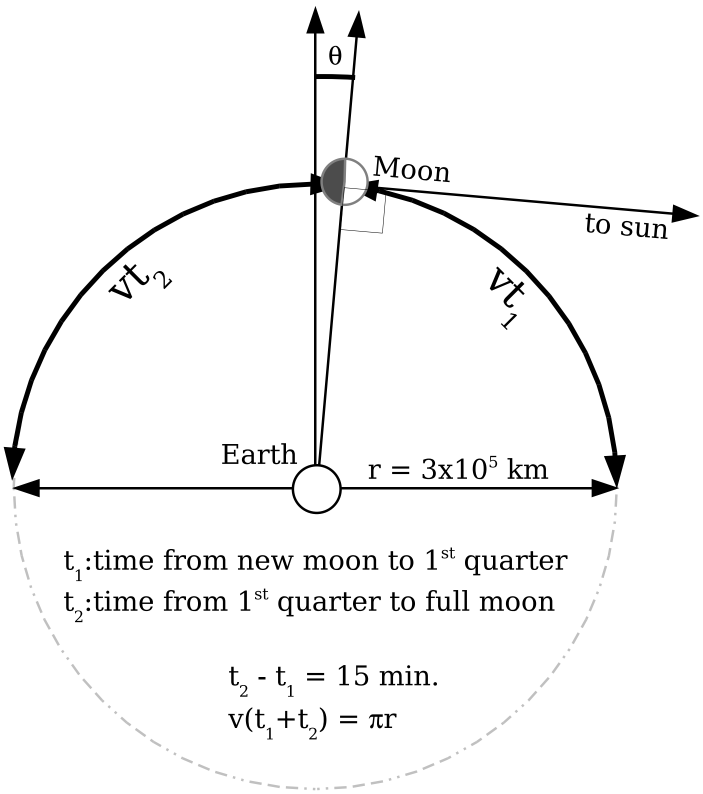

Figure 3: Geometry of the problem used to determine the distance to the sun, as well as the size of the sun, first proposed by the ancient Greek astronomer Aristarchus. The box denotes a right angle.

(a) Given the circumference of the above orbit, what is the speed v of the moon's revolution about Earth? The velocity of the moon in its circular orbit:

$$v = \frac{2\pi r}{P} = \frac{2\pi \times (3 \times 10^{10}\ \text{s})}{86400 \times 28\ \text{s}} = 1.24 \times 10^4\ \text{cm s}^{-1}$$

(b) Use the above system of equations in this figure to solve for t_1 and t_2. Solving the above system of equations given the velocity gives the following result:

$$t_1 = 7\ \text{days} - 7.5\ \text{min} = 6.0435 \times 10^5\ \text{s}$$
$$t_2 = 7\ \text{days} + 7.5\ \text{min} = 6.0535 \times 10^5\ \text{s}$$

Here, due to the smallness of the angle θ, it is important to keep the time measurement to a high degree of accuracy!

(c) What is the angle θ in the above diagram (hint: calculate out the time t such that $vt = r\pi/2$ and compare to vt_1. From the expression $vt = r\pi/2$ implies that $t = 7$ days. The angle θ is then given by:

$$\theta = \frac{vt - vt_1}{r} = \left(1 - \frac{t_1}{t}\right)\frac{\pi}{2} = \frac{7.5\ \text{min}}{7\ \text{days}} \times \frac{\pi}{2} = 1.17 \times 10^{-3}\ \text{rad}$$

(d) What is the distance to the sun given r and θ (hint: use trigonometric relations between the sides of right triangles, and note that the distance from Earth to the sun is the hypotenuse). The radius of the moon's orbit $r = D\sin\theta \approx D\theta$, where D is the earth-sun separation and $\theta \ll 1$ (hence allowing the above approximation to be made). From the above, we get that $D = 2.57 \times 10^{13}$ cm. This is the earth-sun distance, to within an order of magnitude.

(e) Given that the angular size of the sun is estimated to be $0.5^{\text{ﬃ}}$, what is the diameter of the sun? The sun has a angular diameter of $0.5^{\text{ﬃ}} = 8.7 \times 10^{-3}$ radians. Therefore the diameter of the sun $d = 8.7 \times 10^{-3} D = 2.24 \times 10^{11}$ cm, showing that the sun is far larger than anything in the ancient Greeks' experience.

4. Two stars have a period of 1000 years and a separation of 0.5". The parallax of this system of objects is 0.01". Assume we are seeing the system of stars face-on, and that they have the same mass.

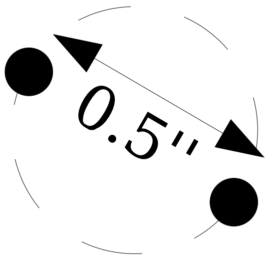

Figure 4: Binary system separated by 0.5" in the sky. The system is assumed to be face-on, and the dashed circle denotes the path across the sky that these objects take.

(a) Can these two stars be separately resolved by a 5 m diameter telescope observing at 500 nm? Use the Rayleigh criterion, that:

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = 1.22 \times 10^{-7} \text{ rad}$$

The angular separation of the stars $\theta = 0.5'' = 2.4 \times 10^{-6}$ rad $> \theta_{\text{min}}$, therefore the stars **can** be resolved.

(b) What is the distance to this binary system, in pc? The parallax of the object is 0.01" (as Earth moves 1 AU transverse to this object). This implies that the system is 100 pc away.

(c) What is the separation between the two stars, and hence the semimajor axis, in AU? Note that an object 1 AU across, seen from 1 pc distance will subtend an angular size of 1". These objects are separated by 0.5", but the objects are 100 pc away. Therefore their separations are $0.5 \times 100 = 50$ AU. Since these objects have the same mass, their center of mass lies between the two objects. The semimajor axis of both objects is 25 AU. The semimajor axis for the reduced mass object is 50 AU.

(d) What are the masses of the two stars? Using Kepler's third law, that $P^2 \propto a^3/M$ (where M is the total mass of the system), we have that:

$$\left(\frac{P}{1 \text{ yr}}\right)^2 = \left(\frac{a}{1 \text{ AU}}\right)^3 \left(\frac{M}{1 \ M_\odot}\right)^{-1}$$

This implies that $M = 50^3/1000^2 M_\odot = 0.125 M_\odot$.

5. A sun-grazing comet has a period of 10^6 years. Its point of closest approach is 8×10^5 km (5.3×10^{-4} AU).

(a) What is semimajor axis a and its maximal distance from the sun, according to Kepler's third law? Kepler's third law implies:

$$\left(\frac{P}{1 \text{ yr}}\right)^2 = \left(\frac{a}{1 \text{ AU}}\right)^3$$

Therefore the semimajor axis $a = (10^6)^{2/3} = 10^4$ AU. The maximal distance $r_{\max} \approx 2a = 2 \times 10^4$ AU.

(b) What is the ratio of speeds at the point of closest approach to the speed at farthest distance? You may use the fact that angular momentum is constant in the comet's orbit. Angular momentum is conserved therefore we have that, where $v_{\max}$ denotes the speed at apogee and $v_{\min}$ denotes the speed at perigee.

$$r_{\max} v_{\max} = r_{\min} v_{\min}$$
$$\frac{v_{\min}}{v_{\max}} = \frac{r_{\max}}{r_{\min}} = \frac{2 \times 10^4}{5.3 \times 10^{-4}} = 3.8 \times 10^7$$

6. Consider the problem of a massless ball bouncing between two walls – a photon. The ball always moves at the speed of light – c. Again consider the case where the walls are moving inwards at speed W. Here, rather than considering the velocity of the particle, we use the *momentum*. The relation between the momentum before the bounce and after the bounce is given by:

$$p_{\text{after}} = p_{\text{before}} \left(1 + \frac{2W}{c}\right)$$

Which is an approximate expression as long as the speed at which the walls are moving inwards $W \ll c$.

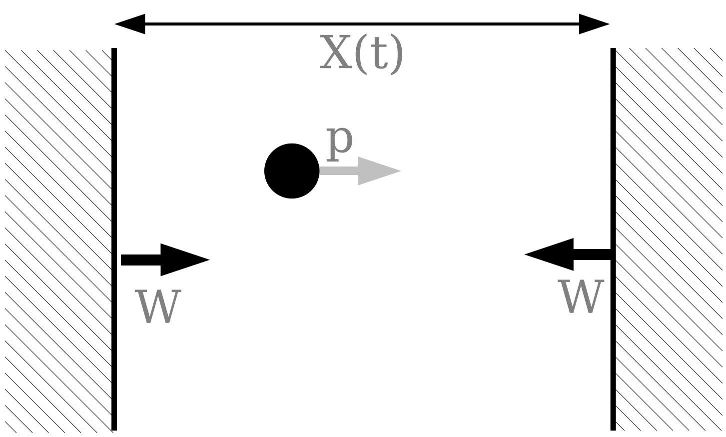

Figure 5: Geometry of a massless particle (photon, for example), bouncing between two walls and acquiring momentum at each bounce. W is the speed of each wall relative to the lab frame, and $X(t)$ is the separation between the walls.

(a) Show that $X(t) = X_0 - 2Wt$, where X is the separation between the two walls. Each wall is moving with velocity $2W$ relative to each other. Therefore $X(t) = X_0 - 2Wt$, where X_0 is the separation at $t = 0$.

(b) Noting that $X = c\Delta t$, where Δt is the time that it takes for the photon to traverse the distance, show that:

$$\frac{dp}{p} = -\frac{2}{c}\frac{dX}{dt}$$

And therefore show that $pX \equiv$ constant. Express $p_{\text{before}} \to p$ and $p_{\text{after}} = p + \Delta p$. The above equation describing the momentum before and after the bounce may then be rewritten as:

$$p + \Delta p = p + \frac{2W}{c} p$$
$$\frac{\Delta p}{p} = \frac{2W}{c} = -\frac{1}{c}\frac{\Delta X}{\delta t}$$

And in the limit of small time steps (or the limit that $c \gg 2W$ we have that:

$$\frac{dp}{dp} = -\frac{1}{c}\frac{dX}{dt}$$

If we note that $c\Delta t \approx X$ (the wall does not change size between bounces, and the particle moves at the speed of light) then the system of equations are given by:

$$\frac{dp}{p} = -\frac{dX}{X}$$
$$d \ln p = -d \ln X$$
$$pX = \text{constant}$$

(c) Consider a cube whose sides are of length L. From a previous problem set, show that $EV^{1/3} =$ constant where E is the energy of a single particle and V is the volume of the box. To solve this problem, assume $p_x X^2 = p_y Y^2 = p_z Z^2$ and use the fact that for a relativistic particle $E = \sqrt{p_x^2 + p_y^2 + p_z^2 c})$. This may be familiar to some as the equation of state of a relativistic gas. Suppose the following relation:

$$p_x = \mathcal{C}/L = \mathcal{C}V^{-1/3}$$
$$p_y = \mathcal{C}/L = \mathcal{C}V^{-1/3}$$
$$p_z = \mathcal{C}/L = \mathcal{C}V^{-1/3}$$

The energy $E = c\sqrt{p_x^2 + p_y^2 + p_z^2} = c\mathcal{C}\sqrt{3}V^{-1/3}$. This implies that $EV^{1/3} \equiv$ constant.

7. A laser puts out 10^{13} erg s^{-1} of power into a circular beam with aperture size of 1 cm. The laser emits light monochromatically at 500 nm.

(a) What is the energy flux (in erg cm^{-2} s^{-1}) and energy density (in erg cm^{-3}) within the beam? The energy flux $\mathcal{F} = P/(\pi R^2) = 3.18 \times 10^{12}$ erg cm^{-2} s^{-1}. The energy density $u = \mathcal{F}/c = 1.06 \times 10^2$ erg cm^{-3}.

(b) Considering the energy of a given photon in the laser beam, what is the flux of photons (photons cm^{-2} s^{-1}) and number density (photons cm^{-3}) at the beam aperture? The energy of each photon $E = hc/\lambda = 3.98 \times 10^{-12}$ erg. Therefore the flux f and number density n of photons:

$$f = \frac{\mathcal{F}}{E} = 8.01 \times 10^{23} \text{ photons cm}^{-2} \text{ s}^{-1}$$
$$n = \frac{u}{E} = 2.67 \times 10^{13} \text{ photons cm}^{-3}$$

(c) Given what you know about diffraction, what is the opening angle of the beam in arcseconds? Using the Rayleigh formula, the opening angle $\theta = \lambda/D = 5 \times 10^{-5}$ rad.

(d) At what radius R from the laser, but within the beam's opening angle, will you have to travel to see one photon cm^{-2} s^{-1}? At large enough distances, the flux of energy goes as:

$$\mathcal{F} = \frac{P}{2\pi R^2 (\theta)^2 / 2} = \frac{P}{\pi R^2 \theta^2}$$

A diagram explaining this in better detail is shown here: Furthermore, the flux of photons from

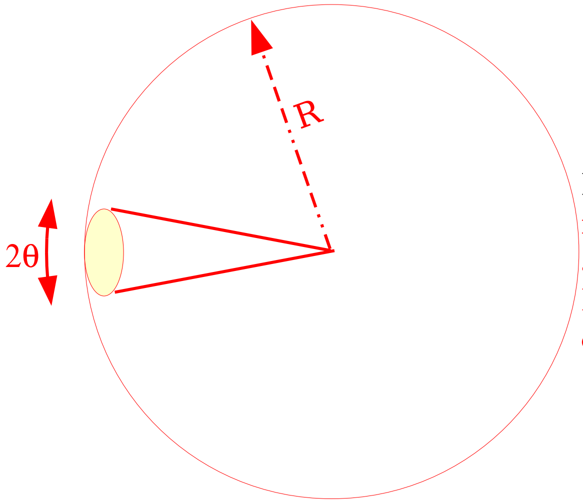

Figure 6: Finite opening angle 2θ of the laser beam, which does not subtend the entire solid angle 4π for a sphere. The solid angle covered by the beam is actually $\Omega = 2\pi (1 - \cos\theta) \approx \pi\theta^2$. Finally, although this is not necessary to know, the intensity falls off as $1/R^2$ when the beam is diffraction limited $(D/R < \lambda/D)$.

the beam as a function of radius from the laser:

$$f = \frac{P}{\pi R^2 (\lambda^2/D^2)} \times \frac{\lambda}{hc} = \frac{PD^2}{\pi R^2 \lambda hc}$$
$$R = \sqrt{\frac{PD^2}{\pi f \lambda hc}}$$

Which results in a distance of $R = 1.8 \times 10^{16}$ cm $\approx 2 \times 10^{-2}$ light-years!

8. A hydrogenic particle has nuclear charge Ze and nuclear mass Am_p. Using the quantization of angular momentum $L = n\hbar$ and by balancing centrifugal forces with electrostatic forces:

(a) What are the radii r_n of the different quantum states with different angular momentum $n\hbar$? What is the radius of the ground state? Use A and Z as undetermined constants. Here we make the simplifying assumption that since the mass of the electron $m_e \ll m_p$, that the system consists of the ion's being stationary. The angular momentum of the system $\ell = n\hbar = m_e v_n r_n$. Solving for the force balance equations:

$$\frac{m_e v_n^2}{r_n} = \frac{Ze^2}{r_n^2}$$
$$m_e^2 v_n^2 r_n^2 = n^2\hbar^2 = m_e Z e^2 r_n$$
$$r_n = \frac{n^2\hbar^2}{m_e e^2} Z^{-1} = n^2 Z^{-1} 5.3 \times 10^{-9} \text{ cm}$$

The radius of the ground state $r_0 = Z^{-1} 5.3 \times 10^{-9}$ cm.

(b) What are the energy levels E_n of the different quantum states; what is the energy of the ground state? You may again use A and Z as undetermined constants. According to the virial theorem, the total energy is half the potential energy. Therefore the total energy:

$$E_n = -\frac{Ze^2}{2r_n} = -\frac{Ze^2}{2} \times \frac{m_e Z e^2}{n^2 \hbar^2} = -\frac{m_e Z^2 e^4}{2\hbar^2 n^2} = -Z^2 \frac{13.6 \text{ eV}}{n^2}$$

So that the energy of the ground state is $-Z^2\,(13.6 \text{ eV})$.

9. Jupiter has a mass $M = 2 \times 10^{30}$ gm and a radius $R = 7 \times 10^9$ cm. Using dimensional arguments, estimate the following:

(a) The central pressure P_c. From dimensional arguments, $P = GM^2/R^4$. Substituting these values for Jupiter yields $P = 1.1 \times 10^{14}$ dyn cm^{-2}.

(b) The average energy per particle. Assume the average particle is a hydrogen atom with mass m_p. The total number of particles $N = M/m_p$. The total thermal energy $\sim$ gravitational binding energy $E = GM^2/R$. Therefore the energy per particle $\epsilon = E/N = GMm_p/R \approx 3.2 \times 10^{-11}$ erg $= 20$ eV.

(c) The average temperature within Jupiter. How does this compare to the outer atmospheric temperature of 200 K? Assume the equation of state is described by an ideal gas. Putting the above expression $\epsilon = k_B T$, given the above energy, $T = 2.3 \times 10^5$ K.

10. A star has density profile $\rho = \rho_0\,(1 - r/R_\star)$, where $R_\star$ is the stellar radius. Calculate the following:

(a) The star mass $M_\star$. Using the mass continuity equation:

$$M_\star = 4\pi\rho_0 \int_0^{R_\star} r^2\,(1 - r/R_\star)\;dr = 4\pi\rho_0 \left(\frac{R_\star^3}{3} - \frac{R_\star^4}{4R_\star}\right) = \frac{1}{3}\pi\rho_0 R_\star^3$$

This implies that $\rho_0 = 3M_\star/\left(\pi R_\star^3\right)$.

(b) The central pressure P_c. The enclosed mass as a function of radius r is given by the following:

$$M(r) = 4\pi\frac{3M_\star}{\pi R_\star^3}\int_0^{r} r'^2\,(1 - r'/R_\star)\;dr' = \frac{12M_\star}{R_\star^3}\left(\frac{1}{3}r^3 - \frac{1}{4R_\star}r^4\right)$$

$$M(r) = 12M_\star\left(\frac{1}{3}\left[\frac{r}{R_\star}\right]^3 - \frac{1}{4}\left[\frac{r}{R_\star}\right]^4\right)$$

The pressure gradient is given by:

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r) = -\frac{36GM_\star^2}{\pi r^2 R_\star^3}\left(\frac{1}{3}\left[r/R_\star\right]^3 - \frac{1}{4}\left[r/R_\star\right]^4\right)(1 - r/R_\star)$$

$$\frac{dP}{dr} = -\frac{36GM_\star^2}{\pi R_\star^5}\left(\frac{1}{3}\left[r/R_\star\right] - \frac{1}{4}\left[r/R_\star\right]^2\right)(1 - r/R_\star)$$

The pressure at the surface is zero, therefore we have that:

$$P\left(R_\star\right) = P_c - \frac{36GM_\star^2}{\pi R_\star^5}\int_0^{R_\star}\left(\frac{1}{3}\left[r/R_\star\right] - \frac{1}{4}\left[r/R_\star\right]^2\right)(1 - r/R_\star)\;dr$$

$$P_c = \frac{36GM_\star^2}{\pi R_\star^4}\int_0^1\left(\frac{1}{3}x - \frac{1}{4}x^2\right)(1 - x)\;dx = \frac{5GM_\star^2}{4\pi R_\star^4}$$

Here we used the variable substitution, $r = xR_\star$, for the last two steps in the integration.

(c) The gravitational binding energy. The gravitational binding energy of this mass distribution:

$$U = \int_V U(\mathbf{r})\,\rho(\mathbf{r})\,dV$$

$$U = -\int_0^{R_\star} \frac{GM(r)}{r}\left(4\pi r^2 \rho(r)\right)\,dr = -4\pi \int_0^{R_\star} GM(r) r \rho(r)\,dr$$

$$U = -4\pi G \frac{3M_\star}{\pi R_\star^3} \times 12 M_\star R_\star^2 \int_0^1 x\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)(1-x)\,dx = -\frac{26GM_\star^2}{35R_\star}$$

Remember, always check that your answers are dimensionally correct.

(d) The average thermal (kinetic) energy per particle. Assume the average mass of a particle is $m_p/2$ (fully ionized hydrogen plasma). According to the virial theorem, the total thermal energy of this system of particles is:

$$K = -\frac{1}{2}U = \frac{13GM_\star^2}{35R_\star}$$

The number of particles is $N = 2M_\star/m_p$, so that the average thermal energy per particle:

$$\epsilon = \frac{2Km_p}{M_\star} = \frac{26GM_\star m_p}{35R_\star}$$

11. Quark stars are theorized to be objects smaller than neutron stars but larger than black holes. These stars have an average particle mass $m_Q > m_p$, where m_p is the proton (approximately neutron) mass. The Schwarzchild radius $R = 2GM/c^2$ is the radius of a black hole's event horizon, and places a lower limit on the size of the object. Here you will attempt to answer questions on the maximum possible particle mass within this object.

(a) Assume a noninteracting nonrelativistic fermi gas. The equation of state is given by:

$$P = 0.0485\frac{h^2 n^{5/3}}{m_Q} = 0.0485\frac{h^2\rho^{5/3}}{m_Q^{8/3}}$$

One can show that the central pressure is given by:

$$P_c = 0.770\frac{GM^2}{R^4}$$

And the central density $\rho_c = 1.43MR^{-3}$. Use these equations to derive a mass-radius relation for the star with m_Q. The equation of state implies the following expression for the central pressure in terms of the central density:

$$P_c = 0.770\frac{GM^2}{R^4} = 0.0485\frac{h^2\rho_c^{5/3}}{m_Q^{8/3}} = 0.0485\,(1.43)^{5/3}\,\frac{h^2}{m_Q^{8/3}}\frac{M^{5/3}}{R^5}$$

$$R = 0.114\frac{h^2}{Gm_Q^{8/3}}M^{-1/3} = 1.54 \times 10^6 \left(\frac{m_Q}{m_p}\right)^{-8/3}\left(\frac{M}{M_\odot}\right)^{-1/3}\ \text{cm}$$

(b) Set $R = 2GM/c^2$. Estimate the maximum value of m_Q, in terms of m_p, for a 1 $M_\odot$ quark star. Given the expression for the radius of a black hole, $R = 2GM/c^2$, we have the following inequality:

$$0.114\frac{h^2}{Gm_Q^{8/3}}M^{-1/3} > \frac{2GM}{c^2}$$

$$m_Q < 0.342M^{-1/2}\left(\frac{hc}{G}\right)^{3/4}$$

$$m_Q < 3.08 \times 10^{-24} \text{ gm}$$

$$\frac{m_Q}{m_p} < 1.8$$

Implying that quark stars must have finely balanced particle masses in order to ensure smaller stars (but large enough that their radii are above the event horizon).

12. A neutrino created in the sun has a cross sectional area of 10^{-41} cm^2 per particle. Here you will answer questions concerning neutrino absorption within the sun.

(a) The mass of the sun $M_\odot = 2 \times 10^{33}$ gm and the radius of the sun $R = 7 \times 10^{10}$ cm. What is the column mass density Σ in gm cm^{-2} for the sun? The column mass density $\Sigma = M_\odot / \left(4\pi R_\odot^2\right) = 3.25 \times 10^{10}$ gm cm^{-2}.

(b) Estimate the average mass per particle $m_p/2$. What is the column number density n_Σ of particles from the surface to the interior in particles cm^{-3}? The column number density then becomes $2\Sigma/m_p = 3.91 \times 10^{34}$ cm^{-2}.

(c) Given the cross section σ, what is the optical depth $\tau = \sigma n_\Sigma$ for neutrino absorption within the sun? Therefore, what fraction of neutrinos that are created within the sun get absorbed? The optical depth to neutrino scattering $\tau = \sigma n_\Sigma = 3.91 \times 10^{-7}$. Since $\tau \ll 1$, only $\tau = 3.91 \times 10^{-7}$ of the neutrinos produced within the sun are absorbed.

13. Here you will estimate the neutrino luminosity (neutrinos cm^{-3} s^{-1}) within the sun's core. The sun's central density $\rho_c = 1.6 \times 10^2$ gm cm^{-3}. 75% of the sun's matter is hydrogen by mass, while 25% of it is helium.

(a) Assuming that He has an atomic number of 4 and atomic mass of 2, what is the average mass per particle within the sun? Electrons do not provide mass. The number density of electrons is given by:

$$n_e = n_{\text{H}} + 2n_{\text{He}}$$

The number densities of hydrogen n_{H} and helium n_{He} are given by:

$$n_{\text{H}} = \frac{3\rho}{4m_p}$$

$$n_{\text{He}} = \frac{\rho/4}{4m_p} = \frac{\rho}{16m_p}$$

Therefore the total number density of particles within fully ionized solar material:

$$n = 2n_{\text{H}} + 3n_{\text{He}} = \left(\frac{3}{2} + \frac{3}{16}\right)\frac{\rho}{m_p} = \frac{27\rho}{16m_p}$$

The average atomic mass per particle $\mu = \rho/n$:

$$\mu = \frac{\rho}{n} = \frac{16}{27} m_p$$

(b) What is the number density n_p of protons within the sun? The number density of hydrogen within the sun:

$$n_{\rm H} = \frac{3\rho}{4m_p} = 7.23 \times 10^{25}\ {\rm cm}^{-3}$$

(c) The neutrino-producing part of the main energy-generating sequence in the proton-proton chain is given by the following:

$${}^1_1{\rm H} + {}^1_1{\rm H} \rightarrow {}^2_1{\rm H} + e^+ + \nu_e \quad \left(1.4 \times 10^{10}\ {\rm yr}\right)$$

The rate at which neutrinos are being created is then given by $\dot{n}_\nu = n_p/\tau$, where τ is the above timescale. What is $\dot{n}_\nu$ in neutrinos cm^{-3} s^{-1}? The rate at which electron neutrinos are produced $\dot{n}_\nu = n_{\rm H}/\tau$. $\tau = 1.4 \times 10^{10}$ yr $= 4.42 \times 10^{17}$ s. Therefore $\dot{n}_\nu = 1.64 \times 10^8$ neutrinos cm^{-3} s^{-1}.