5 6 7 8 9 10 10.95 12 14.4 17.28 20.74 24.88

Practice Problems #2, ASTR 211, Fall 2004

1. The earth is aligned at an angle of 23ﬃ relative to the normal to the ecliptic. The period of Earth's precession is 26,000 years. Assume the North Star is at the North celestial pole ($\delta = 90$ﬃ) at the current epoch. What is the declination of Polaris relative to the old North celestial pole 6,500 years ago? A diagram describing this problem is shown below:

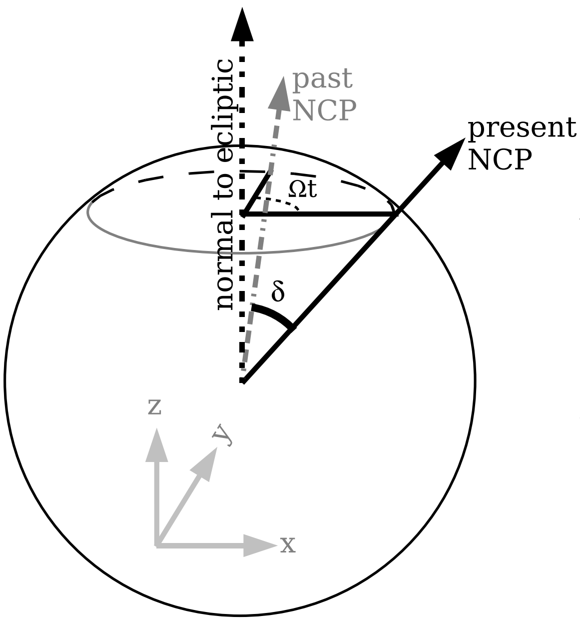

Figure 1: Diagram depicting the vector pointing at the NCP at the current epoch, and at an epoch $t = 6500$ years ago. The angle δ is the angle between the older NCP and the current NCP, or equivalently the angle between the current pole star (assumed to lie at the current NCP) and the old NCP. It is assumed that the heavens are "fixed" relative to the precession of Earth's rotation axis. Axes fixed with respect to the heavens are also shown.

2. Barnard's Star has some of the largest proper motions of any stellar object in the sky. Its radial velocity $v_r = 140$ km s^{-1} towards Earth, while its proper motion across the sky is 10.3″ (10.3 arcseconds) yr^{-1}. It is located 6.7 light-years from the sun.

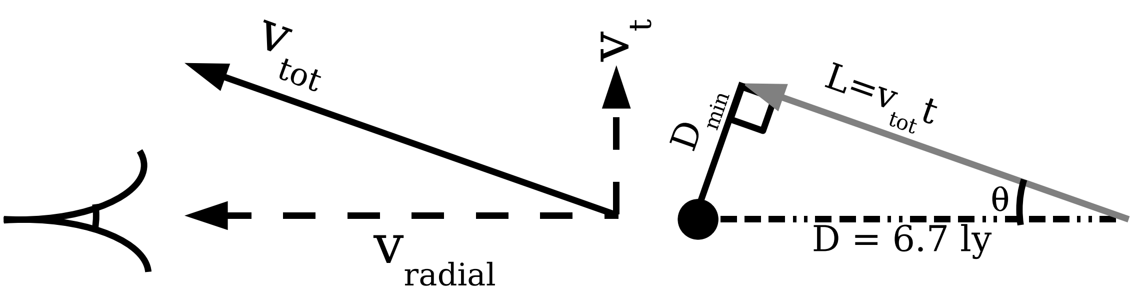

Figure 2: Diagram depicting the velocity and motion (position) of Barnard's star through time.

   (a) What is its transverse velocity v_t? What is the total speed v_{tot} of Barnard's star?

   (b) What is the angle θ that Barnard's velocity vector $\mathbf{v}_{\text{tot}}$ make with our line of sight?

   (c) From the above diagram, what is the closest distance D_{min} that Barnard's star will approach the earth? How many years in the future t will this approach occur?

3. The Greeks observed that the time t_1 from new moon to first quarter is 15 minutes shorter than t_2, the time from first quarter to full moon. Assume the moon has a circular orbit with radius 3×10^5 km and it takes $T = 28$ days to undergo one revolution around Earth.

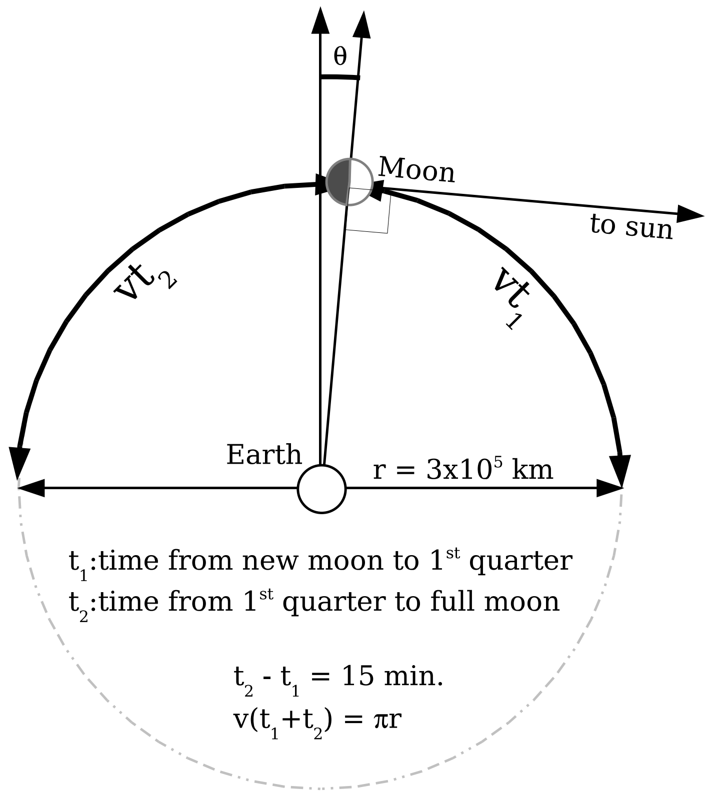

Figure 3: Geometry of the problem used to determine the distance to the sun, as well as the size of the sun, first proposed by the ancient Greek astronomer Aristarchus. The box denotes a right angle.

(a) Given the circumference of the above orbit, what is the speed v of the moon's revolution about Earth?

(b) Use the above system of equations in this figure to solve for t_1 and t_2.

(c) What is the angle θ in the above diagram (hint: calculate out the time t such that $vt = r\pi/2$ and compare to vt_1.

(d) What is the distance to the sun given r and θ (hint: use trigonometric relations between the sides of right triangles, and note that the distance from Earth to the sun is the hypotenuse).

(e) Given that the angular size of the sun is estimated to be 0.5ﬃ, what is the diameter of the sun?

4. Two stars have a period of 1000 years and a separation of 0.5". The parallax of this system of objects is 0.01". Assume we are seeing the system of stars face-on, and that they have the same mass.

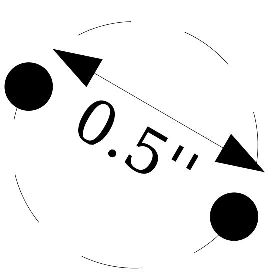

Figure 4: Binary system separated by 0.5" in the sky. The system is assumed to be face-on, and the dashed circle denotes the path across the sky that these objects take.

(a) Can these two stars be separately resolved by a 5 m diameter telescope observing at 500 nm?

(b) What is the distance to this binary system, in pc?

(c) What is the separation between the two stars, and hence the semimajor axis, in AU?

(d) What are the masses of the two stars?

5. A sun-grazing comet has a period of 10^6 years. Its point of closest approach is 8×10^5 km (5.3×10^{-4} AU).

   (a) What is semimajor axis a and its maximal distance from the sun, according to Kepler's third law?

   (b) What is the ratio of speeds at the point of closest approach to the speed at farthest distance? You may use the fact that angular momentum is constant in the comet's orbit.

6. Consider the problem of a massless ball bouncing between two walls – a photon. The ball always moves at the speed of light – c. Again consider the case where the walls are moving inwards at speed W. Here, rather than considering the velocity of the particle, we use the *momentum*. The relation between the momentum before the bounce and after the bounce is given by:

$$p_{\text{after}} = p_{\text{before}} \left(1 + \frac{2W}{c}\right)$$

   Which is an approximate expression as long as the speed at which the walls are moving inwards $W \ll c$.

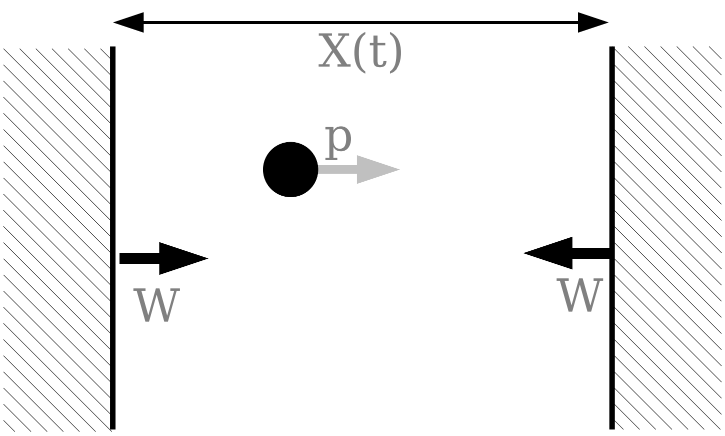

Figure 5: Geometry of a massless particle (photon, for example), bouncing between two walls and acquiring momentum at each bounce. W is the speed of each wall relative to the lab frame, and $X(t)$ is the separation between the walls.

   (a) Show that $X(t) = X_0 - 2Wt$, where X is the separation between the two walls.

   (b) Noting that $X = c\Delta t$, where Δt is the time that it takes for the photon to traverse the distance, show that:

$$\frac{dp}{p} = -\frac{1}{c}\frac{dX}{dt}$$

   And therefore show that $pX \equiv$ constant.

(c) Consider a cube whose sides are of length L. From a previous problem set, show that $EV^{1/3} =$ constant where E is the energy of a single particle and V is the volume of the box. To solve this problem, assume $p_x X^2 = p_y Y^2 = p_z Z^2$ and use the fact that for a relativistic particle $E = \sqrt{p_x^2 + p_y^2 + p_z^2 c})$.

7. A laser puts out 10^{13} erg s^{-1} of power into a circular beam with aperture size of 1 cm. The laser emits light monochromatically at 500 nm.

   (a) What is the energy flux (in erg cm^{-2} s^{-1}) and energy density (in erg cm^{-3}) within the beam?

   (b) Considering the energy of a given photon in the laser beam, what is the flux of photons (photons cm^{-2} s^{-1}) and number density (photons cm^{-3}) at the beam aperture?

   (c) Given what you know about diffraction, what is the opening angle of the beam in arcseconds?

   (d) At what radius R from the laser, but within the beam's opening angle, will you have to travel to see one photon cm^{-2} s^{-1}?

8. A hydrogenic particle has nuclear charge Ze and nuclear mass Am_p. Using the quantization of angular momentum $L = n\hbar$ and by balancing centrifugal forces with electrostatic forces:

   (a) What are the radii r_n of the different quantum states with different angular momentum $n\hbar$? What is the radius of the ground state? Use A and Z as undetermined constants.

   (b) What are the energy levels E_n of the different quantum states; what is the energy of the ground state? You may again use A and Z as undetermined constants.

9. Jupiter has a mass $M = 2 \times 10^{30}$ gm and a radius $R = 7 \times 10^9$ cm. Using dimensional arguments, estimate the following:

   (a) The central pressure P_c.

   (b) The average energy per particle. Assume the average particle is a hydrogen atom with mass m_p.

   (c) The average temperature within Jupiter. How does this compare to the outer atmospheric temperature of 200 K? Assume the equation of state is described by an ideal gas.

10. A star has density profile $\rho = \rho_0 (1 - r/R_\star)$, where $R_\star$ is the stellar radius. Calculate the following:

    (a) The star mass $M_\star$.

    (b) The central pressure P_c.

    (c) The gravitational binding energy.

    (d) The average thermal (kinetic) energy per particle. Assume the average mass of a particle is $m_p/2$ (fully ionized hydrogen plasma).

11. Quark stars are theorized to be objects smaller than neutron stars but larger than black holes. These stars have an average particle mass $m_Q > m_p$, where m_p is the proton (approximately neutron) mass. The Schwarzchild radius $R = 2GM/c^2$ is the radius of a black hole's event horizon, and places a lower limit on the size of the object. Here you will attempt to answer questions on the maximum possible particle mass within this object.

(a) Assume a noninteracting nonrelativistic fermi gas. The equation of state is given by:

$$P = 0.0485 \frac{h^2 n^{5/3}}{m_Q} = 0.0485 \frac{h^2 \rho^{5/3}}{m_Q^{8/3}}$$

One can show that the central pressure is given by:

$$P_c = 0.770 \frac{GM^2}{R^4}$$

And the central density $\rho_c = 1.43 M R^{-3}$. Use these equations to derive a mass-radius relation for the star with m_Q.

(b) Set $R = 2GM/c^2$. Estimate the maximum value of m_Q, in terms of m_p, for a 1 $M_\odot$ quark star.

12. A neutrino created in the sun has a cross sectional area of 10^{-41} cm^2 per particle. Here you will answer questions concerning neutrino absorption within the sun.

   (a) The mass of the sun $M_\odot = 2 \times 10^{33}$ gm and the radius of the sun $R = 7 \times 10^{10}$ cm. What is the column mass density Σ in gm cm^{-2} for the sun?

   (b) Estimate the average mass per particle $m_p/2$. What is the column number density n_Σ of particles from the surface to the interior in particles cm^{-3}?

   (c) Given the cross section σ, what is the optical depth $\tau = \sigma n_\Sigma$ for neutrino absorption within the sun? Therefore, what fraction of neutrinos that are created within the sun get absorbed?

13. Here you will estimate the neutrino luminosity (neutrinos cm^{-3} s^{-1}) within the sun's core. The sun's central density $\rho_c = 1.6 \times 10^2$ gm cm^{-3}. 75% of the sun's matter is hydrogen by mass, while 25% of it is helium.

   (a) Assuming that He has an atomic number of 4 and atomic mass of 2, what is the average mass per particle within the sun?

   (b) What is the number density n_p of protons within the sun?

   (c) The neutrino-producing part of the main energy-generating sequence in the proton-proton chain is given by the following:

   $${}^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + e^+ + \nu_e \quad \left(1.4 \times 10^{10} \text{ yr}\right)$$

   The rate at which neutrinos are being created is then given by $\dot{n}_\nu = n_p/\tau$, where τ is the above timescale. What is $\dot{n}_\nu$ in neutrinos cm^{-3} s^{-1}?