Hints For Problem Set #7

1. The distance modulus is given by

$$m - M = 5 \log_{10} \left(\frac{d}{10 \text{ kpc}} \right)$$

To determine the magnitude of the star, first determine the absolute magnitude of the sun.

2. The magnitude of the star is a measure of the radiant flux of the star as seen from Earth. In other words

$$m_A = -2.5 \log_{10} \left(\tfrac{F_A}{F_{\text{Vega}}} \right), \quad m_B = -2.5 \log_{10} \left(\frac{F_B}{F_{\text{Vega}}} \right)$$

Where F_{Vega} is the radiant flux of Vega as seen from Earth (Vega is a 0 magnitude star). The magnitude of the combined system (assuming no occlusion by one star due to the other)

$$m = -2.5 \log_{10} \left(\frac{F_A + F_B}{F_{\text{Vega}}} \right)$$

You can use the above results to calculate m in terms of m_A *and* m_B. Note that $m < m_A$ and $m < m_B$.

3. There is a difference in magnitudes of 5 between the clusters. Use distance modulus to find the distance to Praesepe.

4. The mistake in the problem lies in their velocities. That is, $v_2 = \Omega r_2$ and $v_1 = \Omega r_1$. Use force balance (centrifugal force balancing out gravitational force) and the relations that $r_1 = m_2 / (m_1 + m_2) r$ and $r_2 = m_1 / (m_1 + m_2) r$ to derive $\Omega^2 = G (m_1 + m_2) / r^3$. The rest of the problem is filled with hints; I do not need to add any more of my own.

5. For 5a, use Kepler's third law, namely that

$$\left(\frac{P}{1 \text{ yr}} \right)^2 = \left(\frac{a}{1 \text{ AU}} \right)^3 \left(\frac{M_{\text{tot}}}{1 \text{ M}_\odot} \right)^{-1}$$

Where M_{tot} is the total mass of the system, P is the orbital period, and a is the separation of the two stars.

For 5b, note that the total energy in the orbit is $E = -GM_1M_2/(2a)$, where a is the separation. Second, that the energy radiated away is energy lost by the system, therefore that:

$$\frac{dE}{dt} = \frac{d}{dt} \left(-\frac{GM_1M_2}{2a} \right) = -\frac{32}{5} \frac{G^4}{a^5 c^5} M_1^2 M_2^2 (M_1 + M_2)$$

And solve the separable differential equation to calculate the lifetime t of this neutron star binary.