Hints For Problem Set #5

1. First calculate the mass of hydrogen in the sun, and multiply by the proportion of that hydrogen that can undergo nuclear burning. Use this mass to calculate out the number of hydrogen nuclei that can undergo burning. Note that each reaction produces energy $0.03 m_p c^2$, and that each reaction uses up 4 hydrogen nuclei; what energy is liberated per hydrogen nuclei. Use that final result to calculate the total energy available for nuclear fusion. Use the solar luminosity $L_\odot$ to calculate the lifetime of the sun, in seconds and years.

2. Note that the probability of an energy generating event in the nucleus is the product of two independent probabilities: 1) the probability that there will be particles moving at relative speed v $\left(\propto \exp\left(-\frac{mv^2}{2kT}\right)\right)$; 2) the probability that two particles with relative velocity v will "tunnel" across their barrier $\left(\propto \exp\left(-\frac{4\pi^2 q_1 q_2}{hv}\right)\right)$. Therefore the probability of interaction within the gas:

$$p \propto \exp\left(-\frac{mv^2}{2kT} - \frac{4\pi^2 q_1 q_2}{hv}\right)$$

   Maximize p with respect to v to find v. Then use this to find the probability p (substitute in v into the above probability expression). Finally, you should get that $p \propto \exp\left(-\left[T_0/T\right]^{1/3}\right)$.

3. Do the following thought experiment – processes that take the products of stellar nucleosynthesis (elements with mass of helium and heavier) and spread them into the interstellar medium must be "explosive." Think about what processes are "explosive" – are they common? Finally, why is it that stars with the mass of the sun cannot recycle substantial amounts of heavier elements into the interstellar medium?

4. Use the following two equations, mass continuity and pressure balance:

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$
$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

   With the following sets of boundary conditions – $M(r=0)=0$, $P(r=0)=P_c$, and $P(r=R)=0$ – and the relation $\rho(r)=\alpha r^{-p}$ to get $P(r)$ and $M(r)$ as well as P_c. From the polynomial expressions you derive, it should be relatively simple to see for what values of the exponent p you will have noninfinite central pressures.

5. Solving this problem is relatively easy. No hints are needed.