Hints For Problem Set #4

1. For part (a), determine the velocity of the ball in the frame of the wall (in a frame where the wall velocity is zero); then calculate out the velocity of the ball after the bounce. You should have your answer for the change in velocity after each bounce.

   For (b), here determine the velocity of the other wall in the frame of one wall. This should give you the relative velocities between walls. Use this to determine the separation of the walls as a function of time.

   For (c), Note that $v_{n+1} = v_n + \Delta v$, so you have an expression for Δv. Assume the velocity of the particle $v \gg A$, the velocity of the wall separation. Then what is Δt, the time taken for the ball to pass from one end to the other? Get an acceleration $\Delta v/\Delta t$, and take the limit as $\Delta t \to 0$ (i.e., $v \to \infty$).

   For (d), suppose that $X = Y = Z = L$. What is the volume of the cube in terms of L, and hence what is L in terms of volume? Assume $v_x X = v_y Y = v_z Z$. The total number of particles in the volume is N. Solve for the kinetic energy in terms of a constant and the volume. Finally, recall that the total energy $E = Ne = \mathcal{E}V$, where e is the energy of a single particle and $\mathcal{E}$ is the energy density.

2. For (a), use the result $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. To solve the problem, what is $\dfrac{d}{dt}\left(\dfrac{1}{2}\dot{\mathbf{r}}^2\right)$, where $\dot{\mathbf{r}} = d\mathbf{r}/dt$?

   For (b), note that the point of closest approach of the two particles is found by letting the total kinetic energy equal the potential energy. Use the form of the potential energy for electrostatic interactions to determine the point of closest approach.

3. For (a), use the virial theorem for r^{-1} potentials to determine the relation between kinetic (thermal), potential, and total energy. Express your result given that $M_\odot = 2 \times 10^{33}$ gm and $R_\odot = 7 \times 10^{10}$ cm, that is, determine $E = \alpha G M_\odot^2 / R_\odot$, where α is a dimensionless constant.

   For (b), use the virial theorem to determine the amount of energy radiated ΔE given a change in potential energy ΔU. Note that power is the time rate of change of energy.

4. Solving this problem is relatively easy. No hints are needed.