Practice Problems: ASTR 211 Final Exam

1. An object in the sky is located at celestial coordinates RA: 19^{h} 23^{m} 14^{s}, Dec: -30^{ffi} 22’ 3”.
   (a) What is the latitude, north of which the object will not be on the horizon?
   (b) At what times of the year will the object be visible in the night sky, where it can be observed? To answer this question, consider how the sun moves across the sky, and assume that the object is “visible” when the sun has a sidereal coordinate 6 hours ahead or behind this celestial object. Assume you are at zero degrees longitude.
   (c) Assume you are in the Northern Hemisphere. On what date will the object have the highest elevation above the horizon at exactly local (solar) midnight? Assume you are at zero degrees longitude.
2. The Zodiac consists of those constellations through which the sun passes through in its yearly motion across the sky. Equivalently, these are the constellations that lie on the ecliptic plane. The Zodiac constellations, and hence the signs that you see in horoscopes, were “finalized” 2500 years ago. Why is it that the Zodiac constellations no longer correspond to a specific Gregorian month (i.e., the sun is no longer in the constellation Leo in the month of August).
3. The sun undergoes circular motion about the galactic center.
   (a) The period of the sun’s orbit is 250 million years and the radius of the sun’s orbit is 10 kiloparsecs. What is the sun’s speed in its orbit around the galactic center?
   (b) What is the mass interior to the sun’s orbit?
   (c) Define a “disruptive” event (or collision) when a star comes within 100 AU of the sun. Given that the density of stars is 0.1 pc^{-3}, what is the probability of a disruptive event within one 250 million year period?
4. An object from Earth is observed to have a synodic period of 2 years.
   (a) What are the **two** possible orbital periods of the star?
   (b) Using Kepler’s third law, what are the two possible orbital radii of these planets?
5. What are the dimensions of the following physical constants? You may express your answer in terms of fundamentals ($M^\alpha L^\beta T^\gamma$, where M is mass, L is length, and T is time) or in terms of cgs units (gm^α cm^β s^γ).
   (a) h (Planck’s constant):
   (b) G (Gravitational constant):
   (c) c (speed of light):
6. A sun-grazing comet has a period of 10^6 years. The point of closest approach of the comet is 800,000 km (5.3×10^{-4} AU).
   (a) What is the semimajor axis of the comet, using Kepler’s third law?
   (b) What is the maximal distance of the comet from the sun, in AU?

(c) Using conservation of total (kinetic + potential) energy and conservation of angular momentum, what is the ratio of kinetic energy at perihilion (closest approach) to aphelion (furthest approach)?

7. A certain binary star system has a period of 50 years. The binary system has parallax of 0.01" and semimajor axis of 0.3".

   (a) What is the semimajor axis, in AU, of this binary system?

   (b) What is the total mass, in $M_\odot$, of the two objects?

   (c) Companion A has semimajor axis of 0.05" as seen from Earth while companion B has semimajor axis of 0.25". What are their masses, in $M_\odot$?

8. A telescope has diameter D, and it collects light of wavelength λ. Derive, using the uncertainty principle, the angular resolution of this telescope.

9. Consider the light from two binary stars. One star has absolute magnitude $M_A = -2.5$, and the other star has absolute magnitude $M_B = 0$.

   (a) What is the absolute visual magnitude of the binary system?

   (b) The stars are located 100 pc away. What is the apparent magnitude of each star and the binary as a whole?

10. A star is observed to have a parallax of 0.2", a proper motion of 1.3"/year, and a Doppler blueshift of 1.1 Åfor a 5500 Å absorption line.

    (a) What is the radial velocity of this star? Is it moving towards or away from us?

    (b) What is the transverse velocity of this star?

    (c) What is the star's total speed?

    (d) **bonus** Assuming the star's relative velocity to the sun does not change, what is the closest approach of the star to the sun? How many years will it take for the star to approach the sun?

11. Here are some questions about photons:

    (a) What is the momentum carried by an individual photon of frequency ν?

    (b) Given a radiation flux of 30 erg s^{-1} cm^{-2}, what is the pressure, in dyne cm^{-2}?

    (c) What is the force acting on a spherical dust grain of radius 10^{-4} cm?

12. A hydrogen like atom has nuclear charge Ze and nuclear mass Am_H. Using the quantization of angular momentum ($L = n\hbar$) and the balance of centrigugal forces with electrostatic forces:

    (a) Calculate the radii of the different quantum states. What is the radius r_0 of the ground state? You may use A and Z as undetermined constants.

    (b) Calculate the energy levels of the different quantum states. What is the energy E_0 of the ground state? You may use A and Z as undetermined constants.

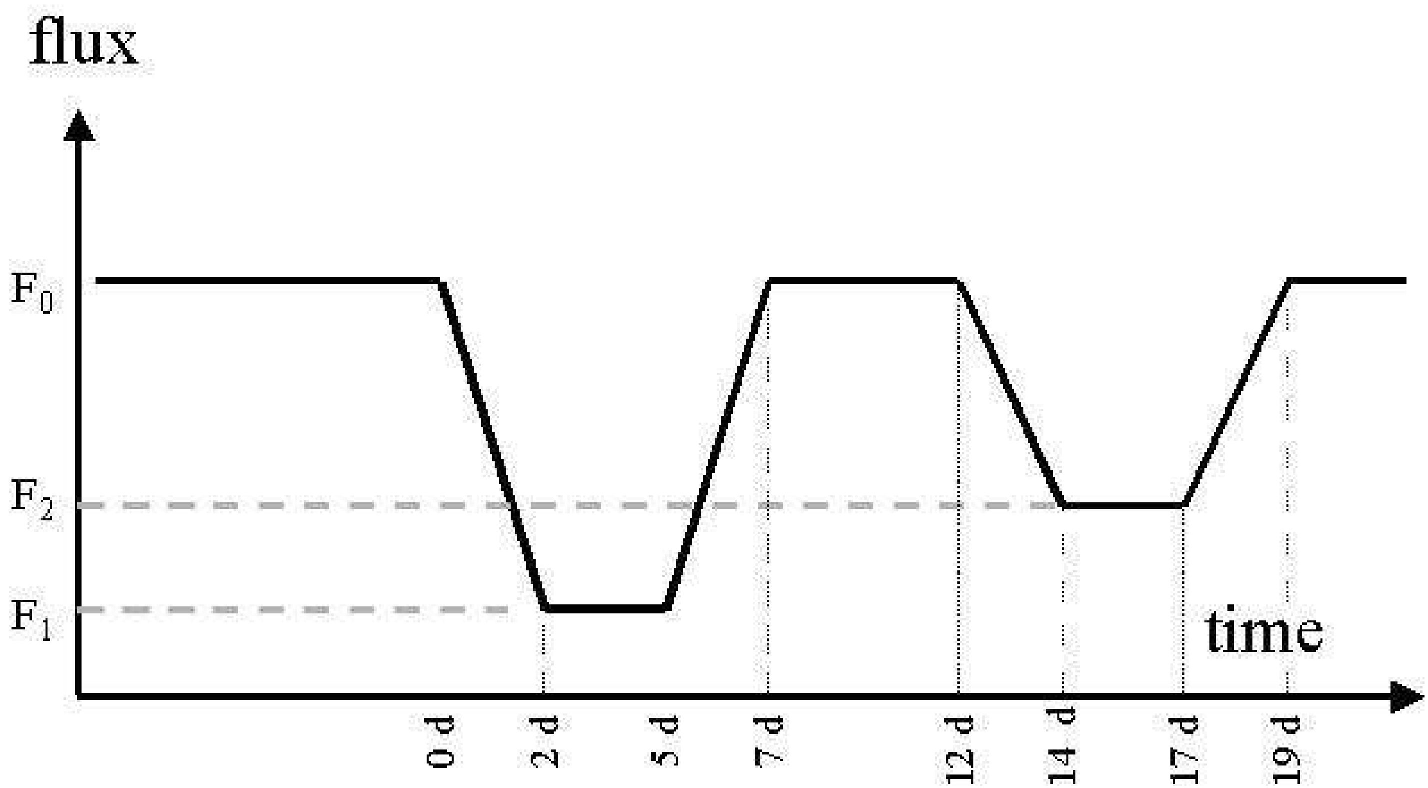
    You may assume the electron mass is much smaller than the nuclear mass.

13. Some interstellar dust has a visual wavelength opacity of 1 cm^2 gm^{-1} and a density of 10^{-18} gm cm^{-3}.

    (a) What is the optical depth assuming a thickness of 1 light year? What is the radiation flux after 1 light year assuming an incident intensity of 10^{-12} erg cm^{-2} s^{-1}?

    (b) What is the visual extinction, in magnitudes per light year, of this dusty cloud? If a star suffers visual extinction $A_V = 2$, what is the physical depth of this interstellar cloud?

14. The following edge-on binary has the following light curve, in visual magnitude. Assume the two stars have identical masses and move in circular orbits.



Let star A has smaller radius than star B. The stars move with velocity 10^7 cm s^{-1} about their center of mass at an inclination of 90ffi.

    (a) What are the radii of star A and star B given the light curves shown above?

    (b) Assuming star A has twice the effective temperature as star B, what is F_1/F_0 and F_2/F_0 for the above stellar system?

15. A star of mass $M_\star$ has density profile given by $\rho = \rho_0\,(1 - r/R_\star)$, where $R_\star$ is the stellar radius.

    (a) What is ρ_0 in terms of $M_\star$ and $R_\star$? Use the mass continuity equation:

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

(b) What is the pressure $P(r)$ as a function of radius, setting $P(r = R_\star) = 0$? Write your answer in terms of $M_\star$ and $R_\star$. Recall the equation for hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

(c) Now replace $R_\star$ with $R_\odot = 7 \times 10^{10}$ cm and $M_\star$ with $M_\odot = 2 \times 10^{33}$ gm. What is the central pressure?

16. Neutrinos have a cross section for capture and interaction of approximately 10^{-41} cm^2 per particle.

(a) Given that the mass of the sun is $M_\odot = 2 \times 10^{33}$ gm, the radius $R_\odot = 7 \times 10^{10}$ cm, what is the column mass density of particles in a column from the sun's surface to the sun's core?

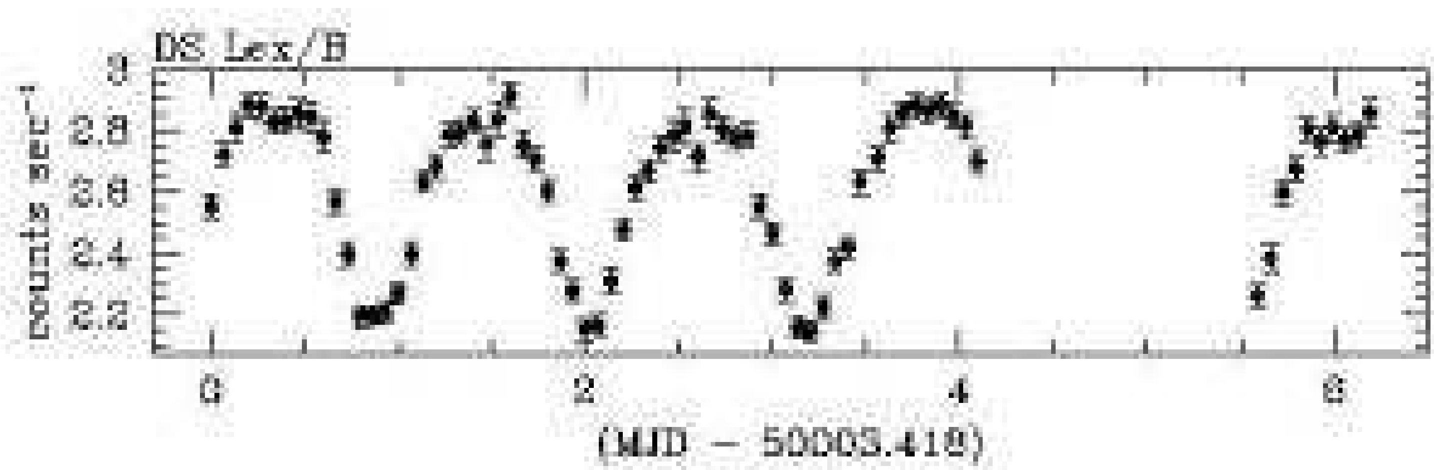
(b) The average atomic mass of material in the sun is 2.2×10^{-24} gm/particle. What is the corresponding number density of particles in a column from the sun's surface to the sun's core?

(c) What is the optical depth of the sun to neutrinos?

17. Why are the angular resolutions of single-dish telescopes, say with a diameter of 100 meters, observing 1 cm wavelength sources, so much worse than optical telescopes, with diameters of 1 m, observing 5000 Å wavelength sources?

18. Describe the main mechanism (κ) that drives stellar pulsation. What is the ϵ mechanism, and why is it unimportant for stellar pulsation?

19. Shown below is the light curve of a typical white dwarf. This variability has period of 1.15 days.



The sound crossing time within a white dwarf is approximately 10 seconds. Is the pulsation due to pressure variations? Why or why not?

20. A perfectly conducting interstellar cloud is threaded with a magnetic field.

(a) How does the magnetic field and magnetic pressure scale with the size R of the cloud, assuming that magnetic flux is frozen?

(b) If we assume adiabatic collapse of the cloud, such that $P \propto \rho^{5/3}$, where ρ is the cloud density, how does the pressure scale with the size R of the cloud?

(c) Based on what you have found, will magnetic fields eventually halt the free-fall collapse of this cloud? Why or why not?

21. From the equation of state of a relativistic degenerate Fermi gas, $P = K\rho^{4/3}$, show that these stars have a fixed mass and undefined radius – that is, show that the mass is given by a constant value independent of the radius of the star. Take K to be something dependent only on physical constants (Planck's constant, the speed of light, etc.) and independent of the star's properties.

22. What gives rise to the Gamow peak in nuclear reactions? Why is quantum mechanical tunneling required to explain the reaction rate in stars? The height of the potential barrier is approximately 50 MeV, and the plasma temperature $T \sim 10^7$ K.

23. The solar wind has a density of 10 protons cm^{-3} and an average speed of 500 km s^{-1} at 1 AU.

    (a) What is the proton flux at 1 AU? Your answer should be in units of cm^{-2} s^{-1}.

    (b) What is the mass loss rate, in $M_\odot$ yr^{-1}, due to the solar wind?

    (c) If we assume that the particle velocity does not change as we change distances from the sun, how does number density scale with d, the distance to the sun?

24. The sun has mass of $M_\odot = 2 \times 10^{33}$ gm and a radius of $R_\odot = 7 \times 10^{10}$ cm.

    (a) What is the gravitational acceleration at the sun's surface?

    (b) What is the maximum angular frequency Ω at which the star will break up? That is, find the Ω such that the centrifugal acceleration balances out the gravitational acceleration.

    (c) What period does this correspond to?

25. A proton, with rest mass of 980 MeV,is travelling at 0.99999 c, where c is the speed of light, relative to an observer on Earth.

    (a) What is the total energy (kinetic + rest mass) of the photon as seen from Earth?

    (b) What is the momentum of the photon as seen from Earth?

    (c) The sun emits radiation at a characteristic wavelength of 5000 Å. What is the wavelength and energy of this photon in the rest frame of the proton? Assume that the photon arrives head-on to the photon.

26. The energy levels of the neutral hydrogen atom go as $E_n = -13.6$ eV$/n^2$. The degeneracy of the energy levels goes as $2n^2$. At $T = 1000$ K:

    (a) What are the relative occupancies of the $n = 2$ and $n = 3$ levels relative to $n = 1$?

    (b) Assuming the density of hydrogen is $\rho = 10^{-3}$ gm cm^{-3}. What are number densities of hydrogen in the neutral state, in the $n = 1$ state, and in the $n = 2$ state?

27. Consider the following about degenerate gases:

    (a) Using the uncertainty principle, show that the average energy of a particle in a nonrelativistic degenerate gas is given by the following: $E \sim \frac{\hbar^2 n^{2/3}}{2m}$, where n is the particle number density and m is the particle mass.

    (b) Show that the pressure in a nonrelativistic degenerate gas goes as $n^{5/3}$.

(c) Using the above expression for the energy, find the critical number density n at which a degenerate electron gas becomes relativistic.

28. Consider the opacity of two fully radiative stars. The opacity of star B is 10 times larger than the opacity of star A.

    (a) If it takes a time τ for photons to radiate out of star A, how many times τ does it take for photons to radiate out of star B?

    (b) Assuming both stars have the same internal energy (same radius, same mass, and same internal structure). How much larger is the luminosity of star A relative to star B? Use dimensional analysis to determine the answer.